

Bifurcations inside Synchronization Regime of Coupled Chaotic Neurons in CPG

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Abstract

Using a model of the heartbeat CPG in the leech we show that cooperative behavior of a network is more regular and reliable than the behavior of individual neurons. We showed that the bifurcations inside synchronization regime are the same as for two coupled model neurons. The synchronization regime exists in a wider region for heartbeat network than for a pair of neurons. The frequency regulation is more steep in the network than in pairs.

1 Introduction

Reciprocally inhibitory coupled neural pairs are an important part of any small neural networks like central pattern generators (CPG). As is well known the typical mode of behavior of such a pair is out-of-phase oscillation (see, for example, [Sherman 1994, Skinner 1994]). It is an important issue to find out how these pairs behave inside a CPG. We address this question studying the dynamics of a model of the heartbeat in the leech.

We took into account that each element of this CPG is a spiking-bursting chaotic neuron (see for example [Aihara and Matsumoto 1986, Hayashi and Ishizuka 1992]). For neural pairs the presence of mutually inhibitory coupling suppresses chaos and the model neurons get in out-of-phase [Abarbanel *et al.*, 1996]. In a CPG network we wonder how the modes of behavior are modified as the the strength of the coupling is changed and how stable the synchronization regime remains. We perform this studying the local bifurcation between the different regimes.

We modeled the heartbeat CPG in the leech using Hindmarsh-Rose chaotic oscillators [Rose and Hindmarsh, 1985] wherein we found that all the nearest neighbors oscillate in out-of-phase. As the strength of the coupling is raised we revealed the types of bifurcations that underlie the change of the CPG period.

2 The model for heartbeat CPG in the leech

We consider the heart beat generated by the network shown in Fig. 1 [Calabrese *et al.*, 1995]. The individual neurons are modeled by

$$\begin{aligned} \frac{dx_i}{dt} &= y_i + ax_i^2 - x_i^3 - z_i + I - \epsilon \left(\frac{x_i + V_c}{1 + \exp \frac{x_{i-1} - X}{\sigma}} - \frac{x_i + V_c}{1 + \exp \frac{x_{i+1} - X}{\sigma}} \right) \\ \frac{dy_i}{dt} &= 1 - bx_i^2 - y_i, \\ \frac{dz_i}{dt} &= -rz_i + rS(x_i - c_x), \end{aligned} \quad (1)$$

where x_i is the membrane potential, y_i is the recovery variable and z_i is the slow adaptation current. In our calculation we took $a = 3$, $b = 5$, $c_x = -1.6$ and $S = 4$. $I = 3.281$ is the external current, and $r = 0.0021$ is the slow recovery parameter. For these parameter values each Hindmarsh-Rose model neuron exhibits chaotic behavior. The coupling parameters are: ϵ is the strength of the coupling, $\sigma = 0.01$, $V_c = -1.4$ and $X = 0.8$.

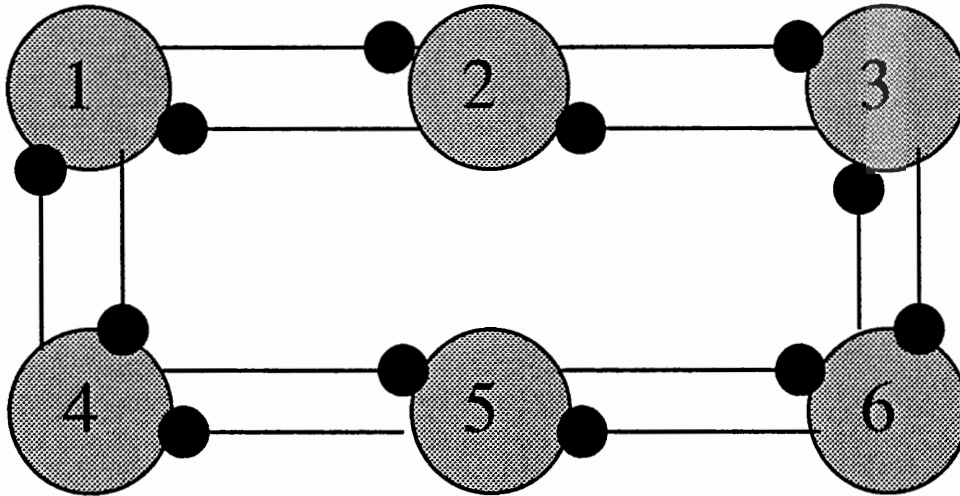


Figure 1: Network diagram that shows the synaptic connectivity between the interneurons of the heartbeat CPG of the leech. The filled circles indicate inhibitory connections. Odd numbers correspond to left cells and even numbers to right cells.

The results of the modeling can be seen in Fig. 2 where the period of the oscillation is plotted as a function of the strength of the coupling in the regime of out-of-phase oscillations. What we find is a staircase change in the period of the oscillation with a particular bifurcation from one step to another one. This frequency regulation represents a collective regularization of the oscillations of the individual neurons in

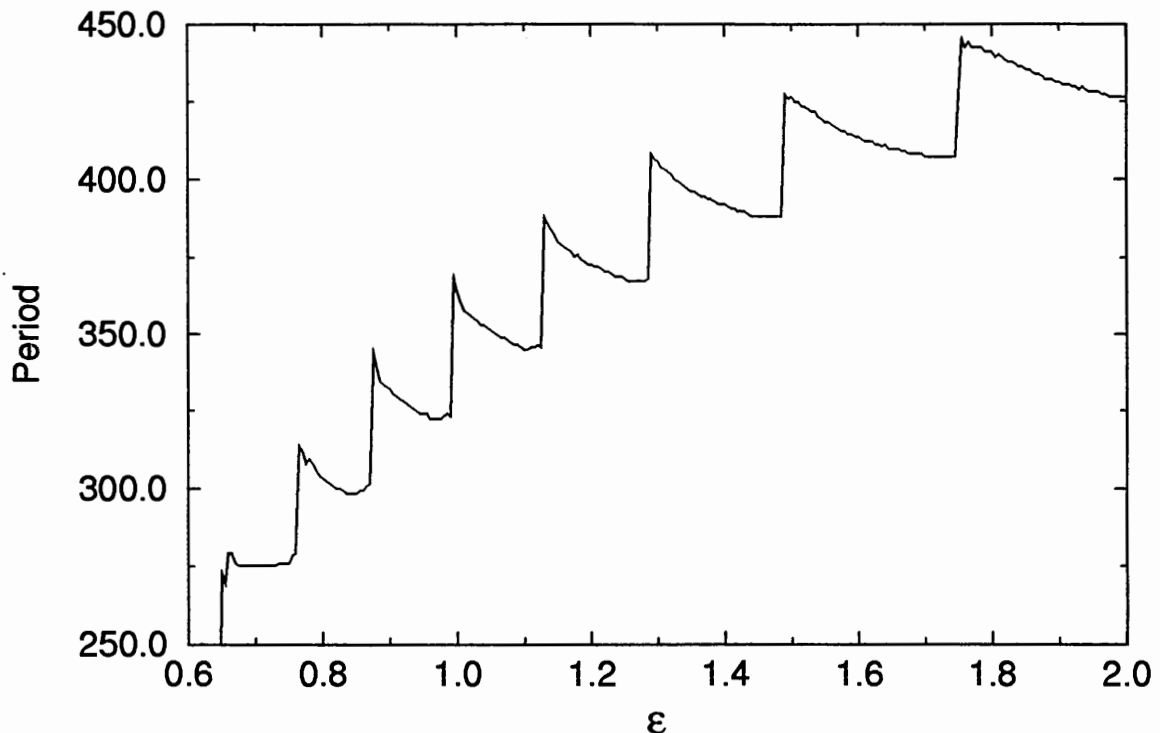


Figure 2: The period of the oscillation generated by a CPG as a function of the strength of the coupling.

the CPG. Left to themselves the individual neurons would be chaotic. Even more interesting is the fact, as displayed in Fig. 3 that nearest neighbor neurons oscillate completely regularly and out-of-phase. This permits the pumping of blood for the leech by alternating voltage signals to the appropriate motor functions from alternate neurons. The leech CPG is able to regulate blood flow to the heart with this assembly of neurons in a robust fashion since the same frequency of pumping is achieved for a range of coupling values among the CPG members. If the synaptic coupling strengths, which we assume are altered by neuromodulators, are not precisely on target, the leech is still able to pump at a selected rate. By varying the coupling, again through the action of neuromodulators, the leech is able to change the pumping rate to meet the need for increased or decreased blood flow presumably in response to sensory signals from external conditions.

3 Bifurcations inside synchronization

Let us consider the behavior of the system of coupled Rose-Hindmarsh cells (see Figure 1) as a function of the strength of the coupling from a mathematical point

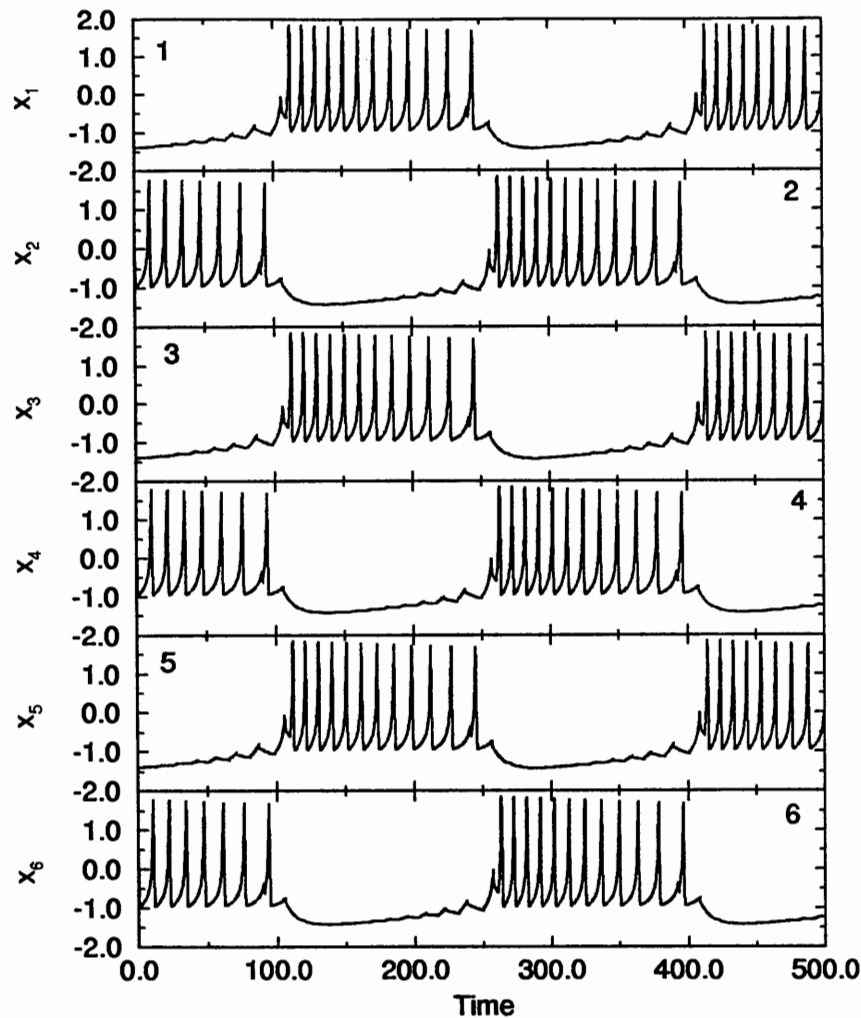


Figure 3: Simulated membrane potential oscillations in the six heartbeat interneurons in the leech. It can be seen that left and right cells oscillate in antiphase.

of view. A periodic orbit in the phase space corresponds to any periodic regime discussed in the previous section. The transition from one periodic regime to another one means that the system “switch” from one periodic orbit to another. The values of the strength of the coupling where it happens are called the bifurcation points.

We will consider these bifurcations using the so-called Poincare map. Let us explain shortly this approach. The trajectories of our system belong to the N -dimensional phase space with coordinates $x_1, y_1, z_1, x_2, \dots, z_N$. The condition $\xi_i = \bar{\xi}_i = Const$ (where ξ_i is any coordinate) determines the $(N - 1)$ -dimensional subspace. Generally the phase trajectories of our system intersect this subspace in some points: $\xi_i = \bar{\xi}_i, \xi_j = \xi_j^{(k)}$, where ξ_j is one of the remaining coordinates and k is the number of the intersections. The Poincare map gives the dependence of the coordi-

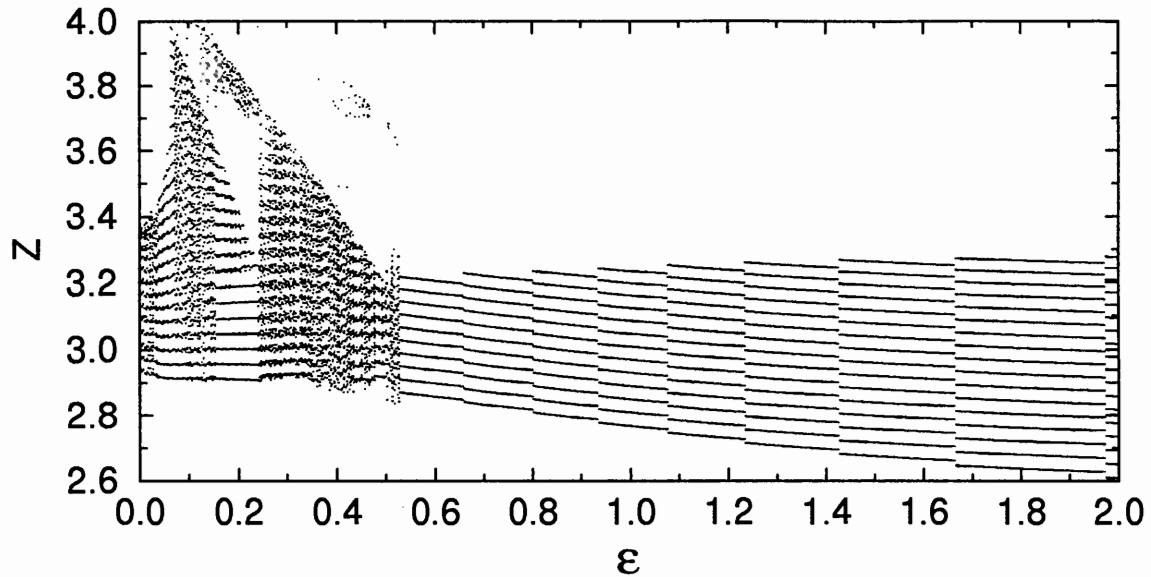


Figure 4: The Poincaré map $x_1 = 0$ for the system of six coupled elements as a function of the inhibitory coupling

nates $\overline{\xi_j^{(k)}}$, $k = 1, 2, \dots$ of these points on some control parameter.

In Fig.4 we present the Poincaré map for the system of six coupled elements as a function of the parameter ϵ . The vertical axis shows the z_1 (z coordinate of the first cell) in the discrete instants of the time: $\dots, t_i, t_{i+1}, \dots$, where t_j - the time instant when the phase trajectory intersects the plane $x_1 = 0$ (x_1 is the membrane potential of the first cell) from the $x_1 < 0$ half space to $x_1 > 0$ half space. It is clear that the system demonstrates periodic behavior with N spikes if the number of these intersections is finite and equals N . If the sequence of the points where the trajectory intersects the Poincaré map is not repeated the trajectory belongs to the strange attractor and the system demonstrates the chaotic behavior.

In Fig.4 we clearly show that for small values of the coupling $\epsilon < \epsilon_{cr}$ the system behavior is generally chaotic. More detailed analysis leads us to find that there exist the narrow intervals of the strength of the coupling ϵ where the system behavior is regular. Inside these intervals the elements demonstrate the "in-phase" synchronized oscillations. The behavior of the system is changed completely for $\epsilon > \epsilon_{cr}$. Starting from the point $\epsilon = \epsilon_{cr}$ the system demonstrates the regular oscillations with "out-of-phase" synchronization between the elements. The increasing of the strength of the coupling for $\epsilon > \epsilon_{cr}$ produces a sequence of bifurcations. In every bifurcation point $\epsilon = \epsilon_N^+$, the periodic orbit with N spikes loses the stability and the system evolves to a periodic orbit with $N + 1$ spikes. If ϵ is decreased then the opposite behavior is observed (in the point $\epsilon = \epsilon_N^-$ the periodic regime with $N + 1$ spikes loses stability). Let us remark that $\epsilon_N^- < \epsilon_N^+$ for all N . That means that in the neighborhood of the bifurcation point the stable periodic orbits with different number of spikes (and

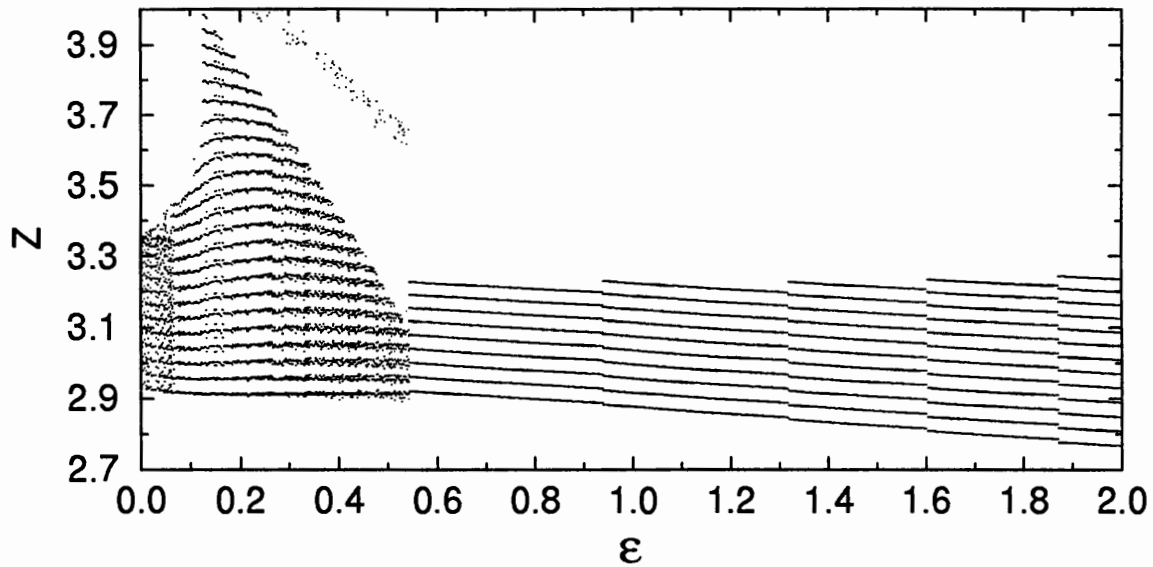


Figure 5: The Poincaré map $x_1 = 0$ for the system of two coupled elements as a function of the inhibitory coupling

consequently with different periods) coexist and the initial conditions determine which regime is realized, that is, an hysteresis process.

We must emphasize that for $\epsilon > \epsilon_{cr}$ the bifurcations do not break the regime out-of-phase synchronization. So it is possible to name such a behavior as a bifurcation inside synchronization.

In Fig.5 we present the Poincaré map for two coupled Rose-Hindmarsh cells. One can see that this system demonstrates a behavior very similar to the one considered above if $\epsilon < \bar{\epsilon}_{cr}$ and exactly the same sequence of bifurcations if $\epsilon > \bar{\epsilon}_{cr}$. The bifurcation points changing the number of the spikes are satisfied by the relation $\bar{\epsilon}_K^{\pm} = (1/2)\bar{\epsilon}_K^{\pm}$. The last is clear if to remember that each element of the loop is connected with two neighbors.

Generally the arbitrary loop of M (where M is even) coupled cells demonstrates the similar sequence of bifurcations. To understand the origin of such a behavior let us consider the dynamics of the isolated Rose-Hindmarsh cell.

The Poincaré map as a function of the external current I for an isolated cell is presented in Fig.6. One can see that increasing of the value of the external current I produce the sequence of the bifurcations. Generally all this bifurcations provide the change of the number of spikes. In the most points such a "switch" takes place directly (without transition to the chaos). In the neighbors of these points the system behavior is very close to the behavior of the coupled cells near the bifurcation points for large coupling $\epsilon > \epsilon_{cr}$. In some cases transition from the regime with N spikes to the regime with $N+1$ spikes occurs more completely. After the first bifurcation where the N -cycle loses the stability the system demonstrates a sequence of bifurcations

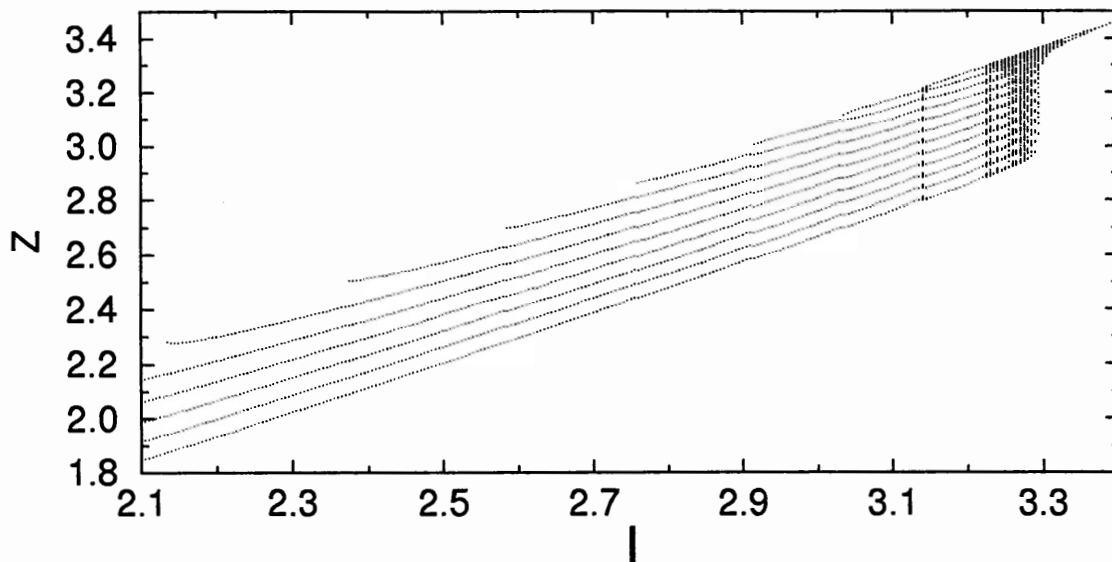


Figure 6: The Poincaré map $x_1 = 0$ of one Rose-Hindmarsh model as a function of the current, I .

as the result of which the chaotic attractor shows up. After, the chaotic attractor disappears and, then, a stable $N + 1$ -cycle arises.

Therefore, one can say that the phenomena demonstrated by the loop of coupled cells have the origin in the dynamics of the isolated neuron. However, generally, the behavior of coupled cells is more regular.

4 Conclusions

The model of the heartbeat CPG of leech that we used as an example of cooperative behavior of spiking-bursting neurons leads us to observe that the behavior of the neuron in the network is more regular (even periodic) than the individual neuron's dynamic. Even chaotic neurons are synchronized and behave in absolutely regular way. We revealed that this synchronized regime is very stable and the changing of the control parameter (strength of the coupling in our case) does not break the regular cooperative behavior but change the period of oscillations. We showed that the bifurcations inside the synchronized behavior of the whole CPG has the same origin that the bifurcations demonstrated by the pair of inhibitory coupled neurons, but with other strength of the coupling $\bar{\epsilon}_K^\pm = (1/2)\bar{\epsilon}_K^\pm$.

The mostly important message that we understand after the study of the whole CPG behavior, the neural pair behavior and individual neuron's dynamic is the following. The adaptivity and flexibility of the CPG dynamics is the result of the complex dynamic of individual neuron. The coupling between neurons plays a role of the

auto-controlling and tuning different modes of behavior depending on the strength of the coupling.

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