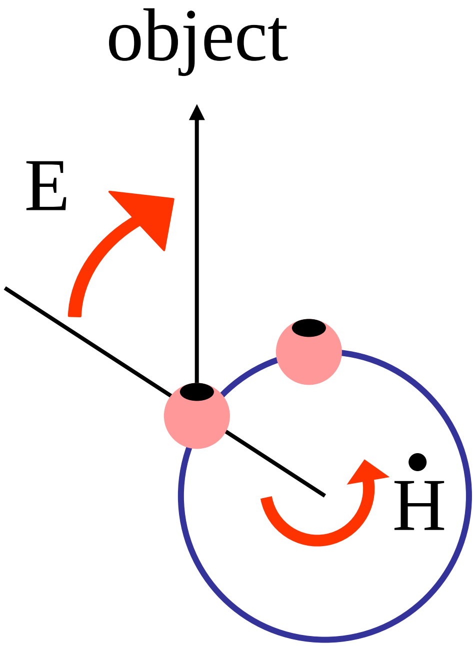
Hysteretic models for a neural integrator

Maxim Nikitchenko

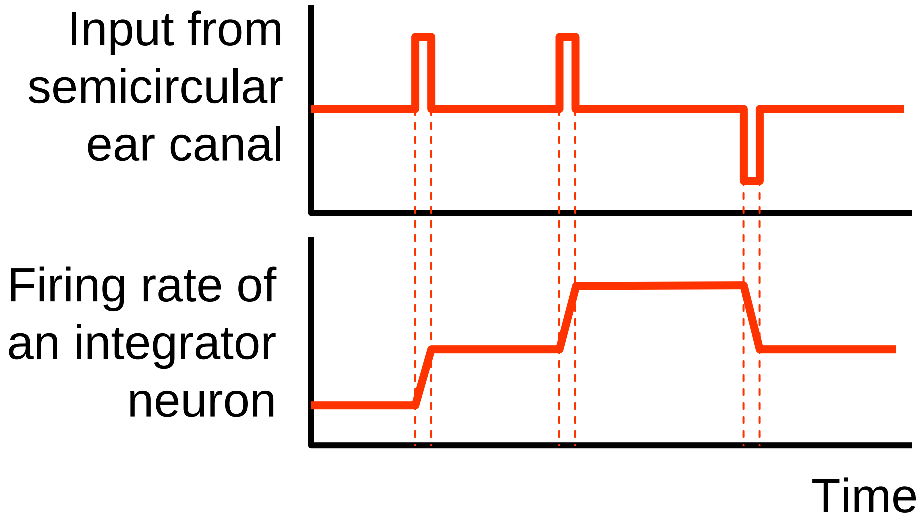
Alex Koulakov

Cold Spring Harbor Laboratory

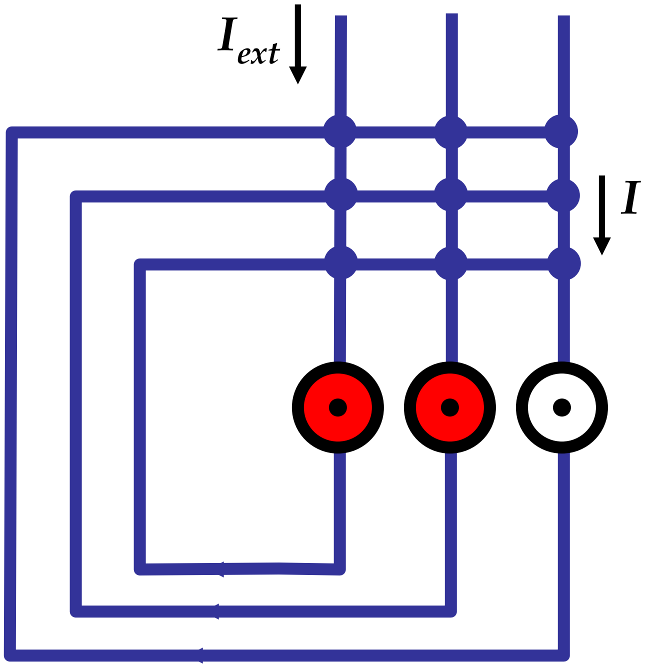
Oculomotor Integrator

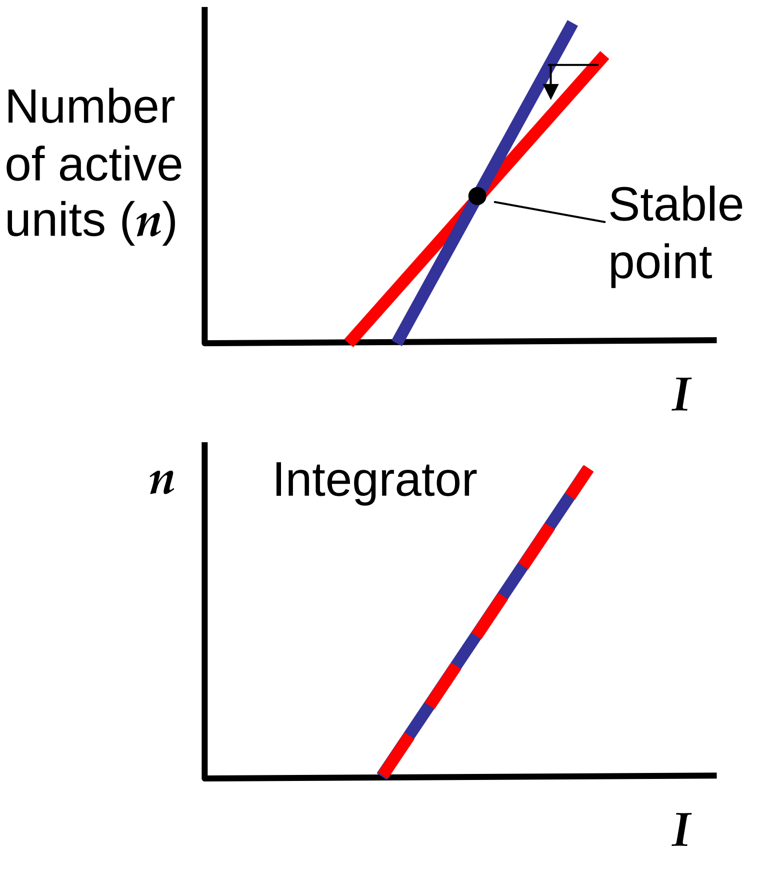


$$E = -\int \dot{H}(t)\,dt$$

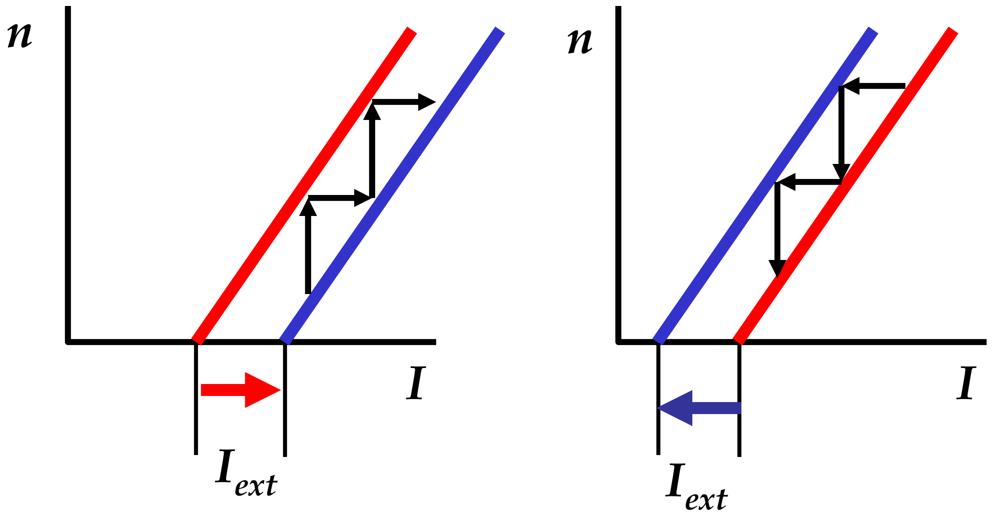


Positive Feedback Mechanism

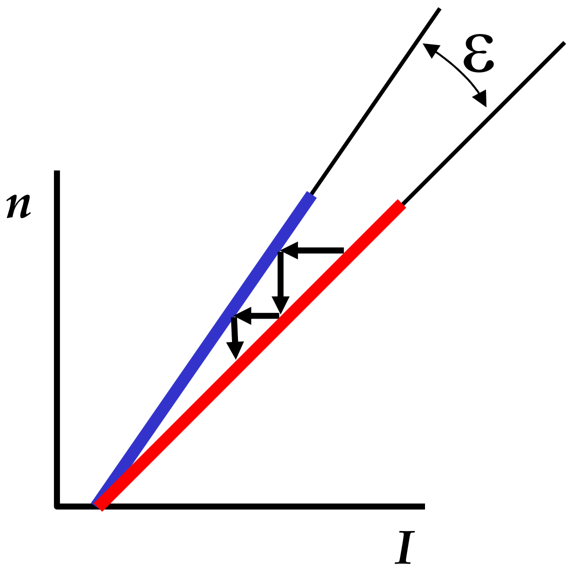




Temporal Integration



Leaky Integrator



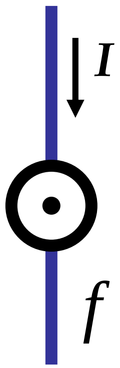
$$\tau = \tau_f N = \tau_f / \varepsilon$$

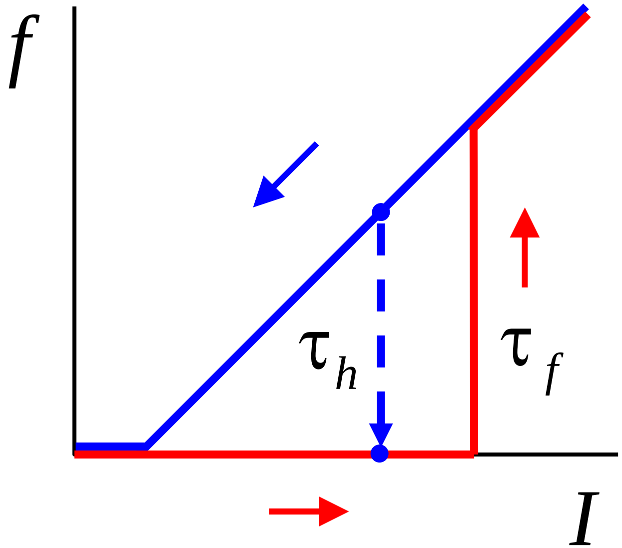
$$\tau_f = 0.1 \sec$$

$$\tau = 30 \sec$$

$$\varepsilon \sim 0.003$$

Hysteretic units solve the problem of robustness



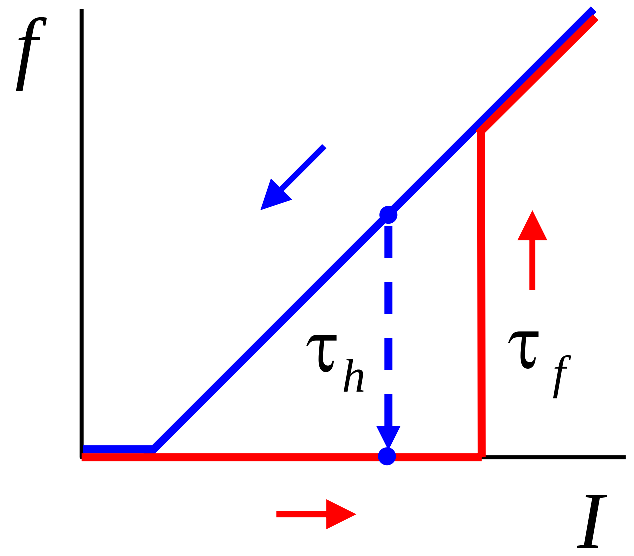


Rosen (1972)
Koulakov, Raghavachari, Kepecs, and Lisman (2002)

Leak time $\tau = \tau_h / \varepsilon$

$\tau_f << \tau_h$

Hysteretic units solve the problem of robustness



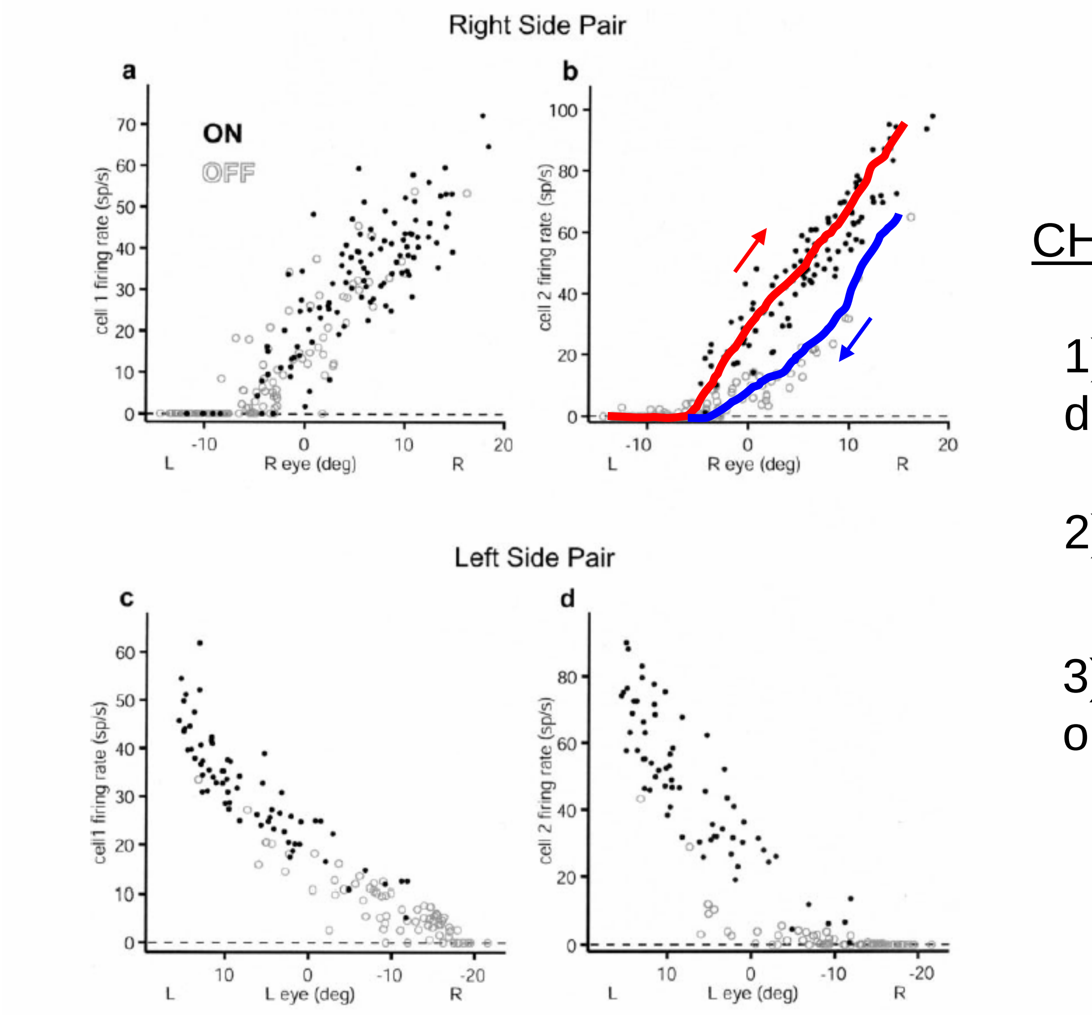
Integration: $\tau_f \sim 0.1\mathrm{sec}$

Memory (leak time): $\tau = \tau_h / \varepsilon$

$$\tau_h = \tau_f \exp\left[C\left(f\tau_f \right) N_{syn} \left(\Delta I / I \right)^3 \right] >> \tau_f$$

Koulakov, 1999

Hysteresis is observed in during fixations in goldfish area I



Aksay, Major, Goldman, Baker, Seung, Tank, *Cerebral Cortex*, 2003

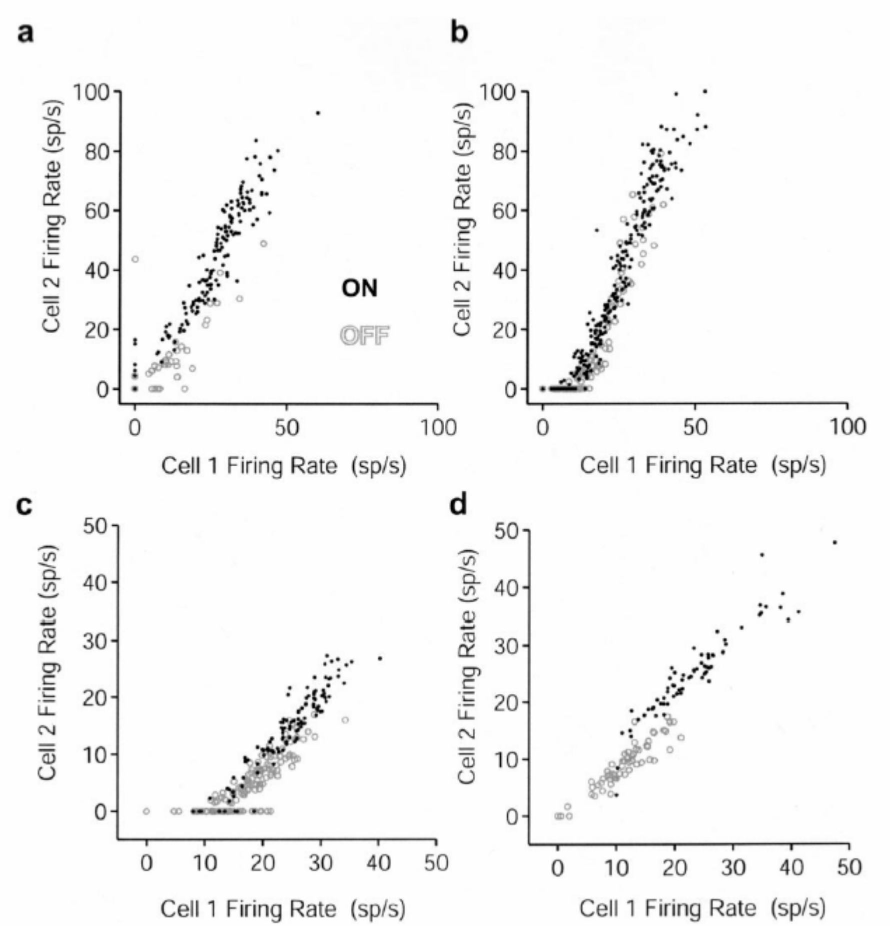
<u>CHALLENGES FOR THEORY:</u>

1) Hysteresis is predicted during VOR (not fixations).

2) Wrong sign of hysteresis

3) Hysteresis in firing of one cell versus the other

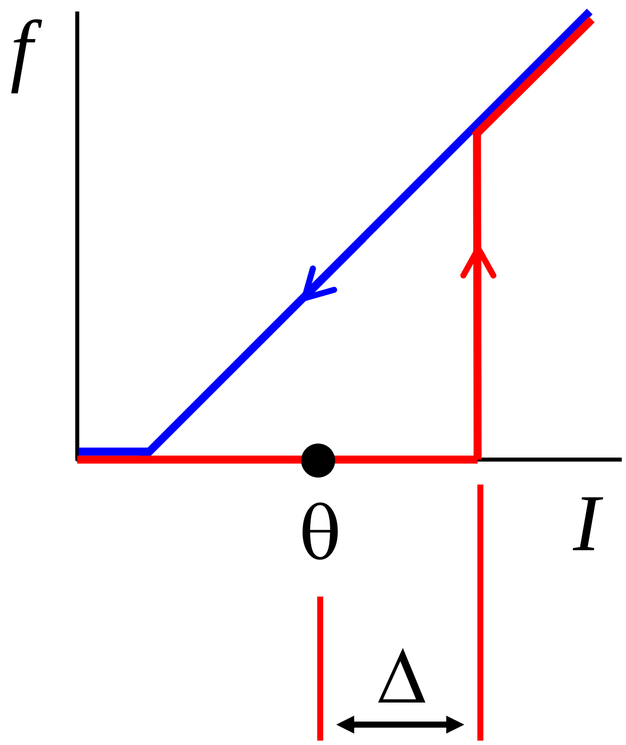
3) Hysteresis in firing of one cell versus the other



Aksay, Major, Goldman, Baker, Seung, Tank, *Cerebral Cortex*, 2003

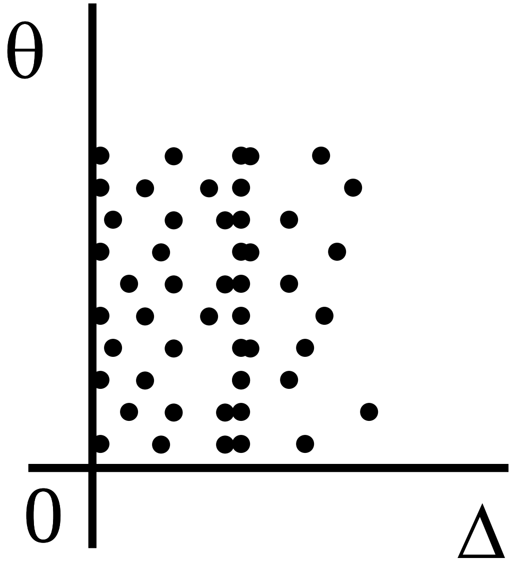
1) Model with differential hysteresis

Model based on inhomogeneous ensemble of hysteretic units



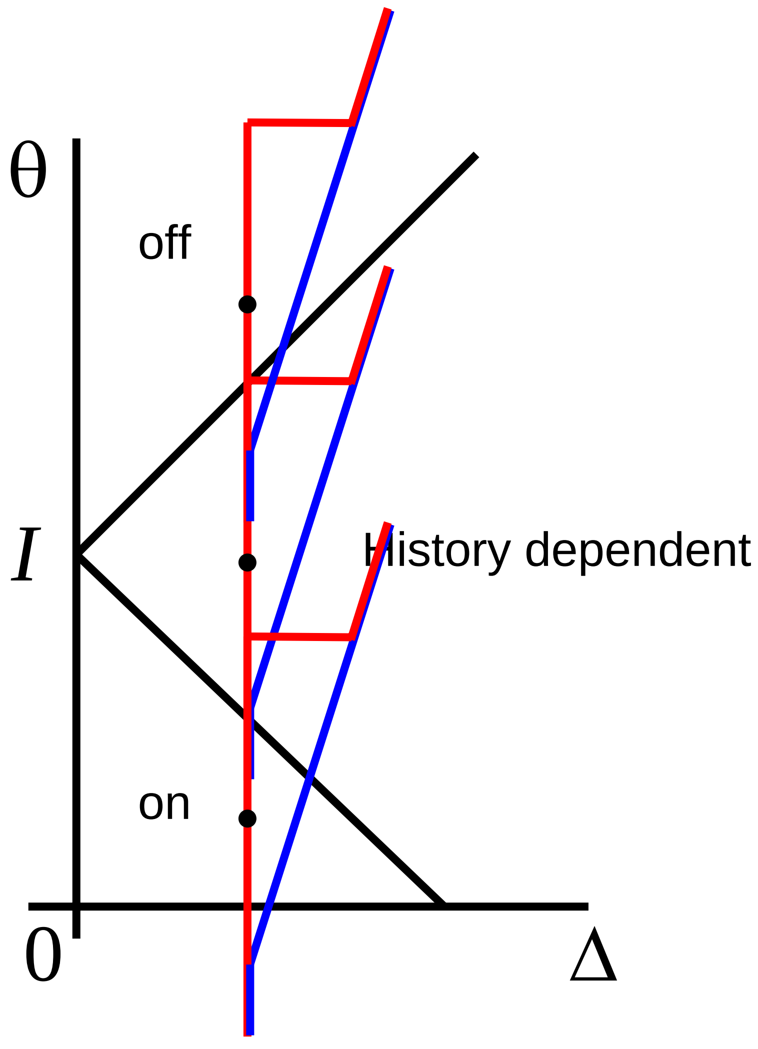
This model

Before (Kondakov et. al., 2002)

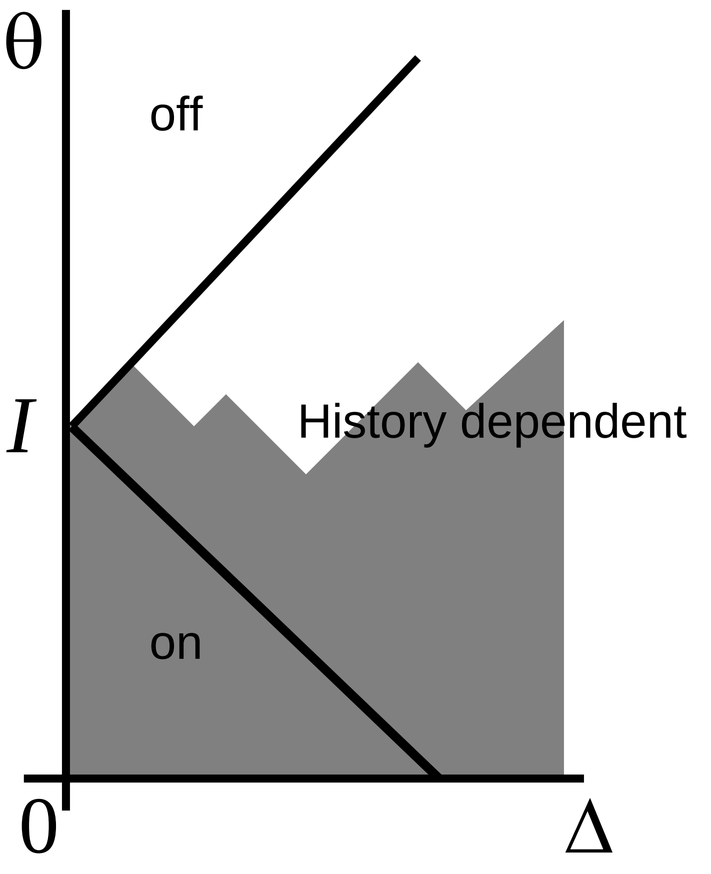


$$\rho(\Delta) = \rho_0 \exp\left(-\Delta / \overline{\Delta}\right)$$

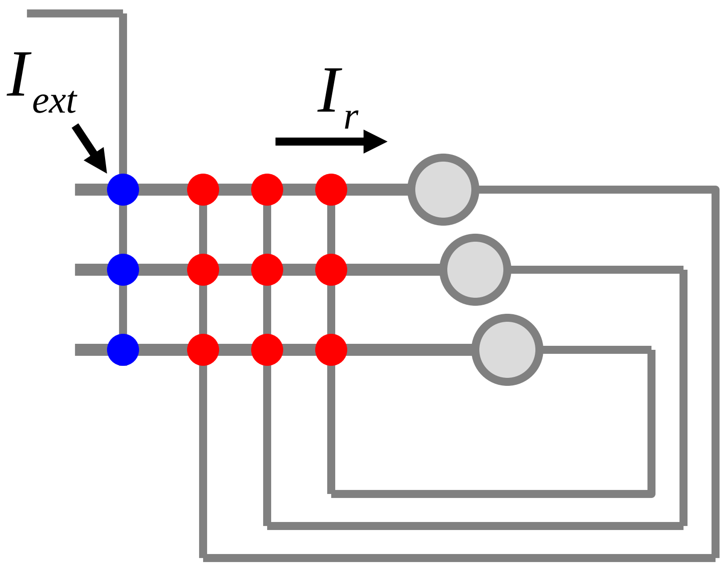
The distribution function of active units is history- dependent



Possible activation profile

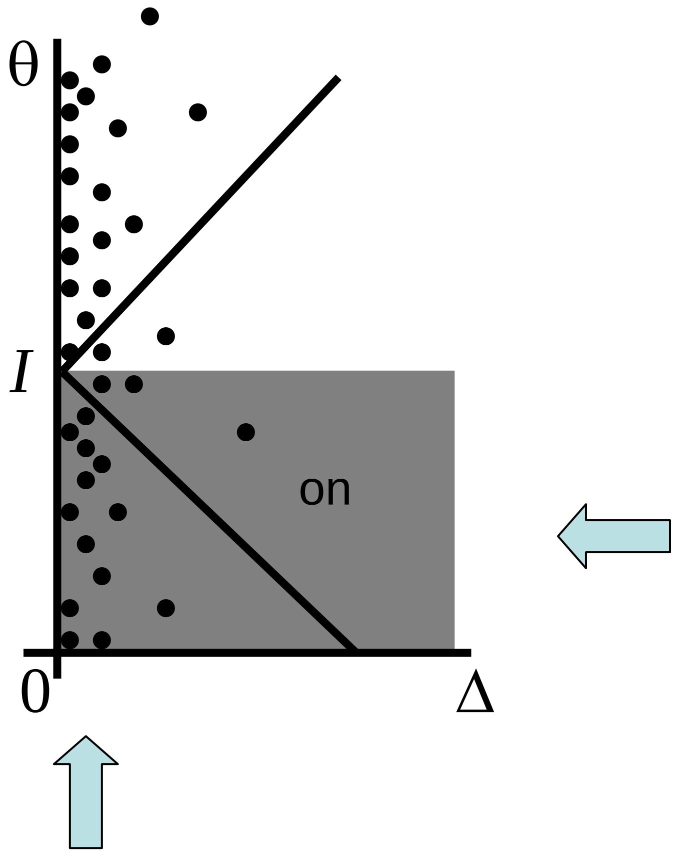


Model with feedback can act as integrator



$$I = I_r + I_{ext}$$

Additional constraint: Stability condition



$$\rho(\Delta) = \rho_0 \exp\left(-\Delta / \overline{\Delta}\right)$$

<u>During fixations:</u>

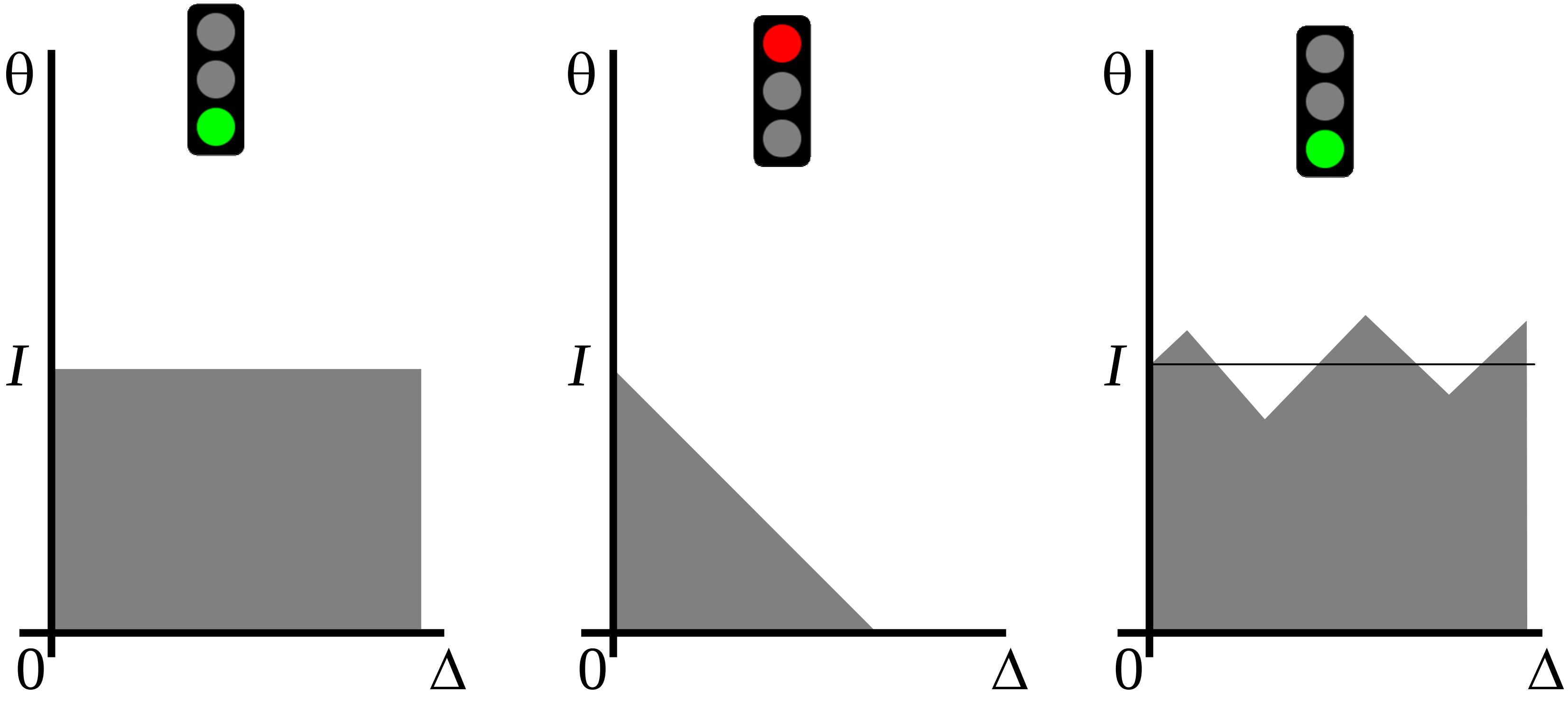
$$I_{ext} = 0$$

$$I = I_r = I_0 n_{on} = I_0 \int f(\Delta, \theta) \rho(\Delta) d\Delta d\theta =$$

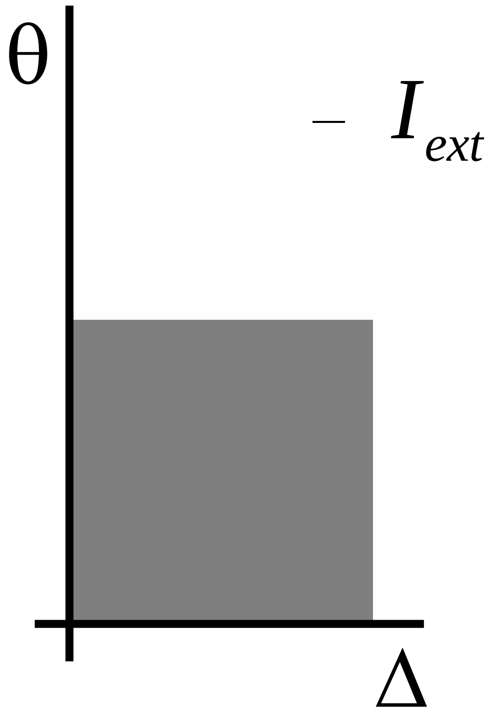
$$= I_0 \rho_0 \overline{\Delta} \cdot I = CI$$

$$f(\Delta, \theta) = \begin{cases} 1, \text{ for ON units} \\ 0, \text{ for OFF units} \end{cases}$$

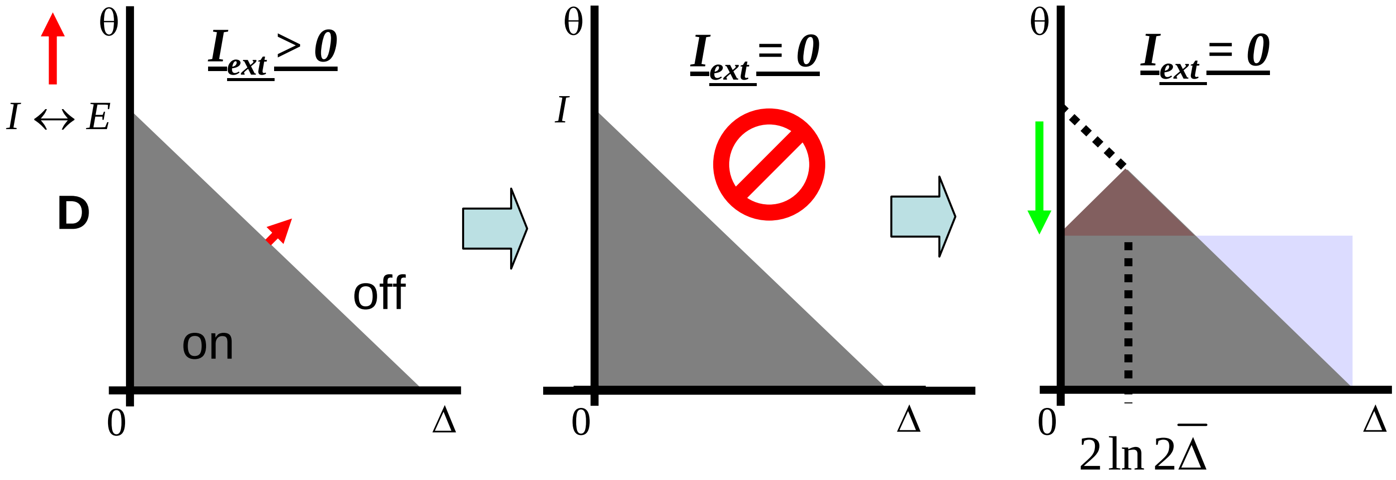
Possible solutions



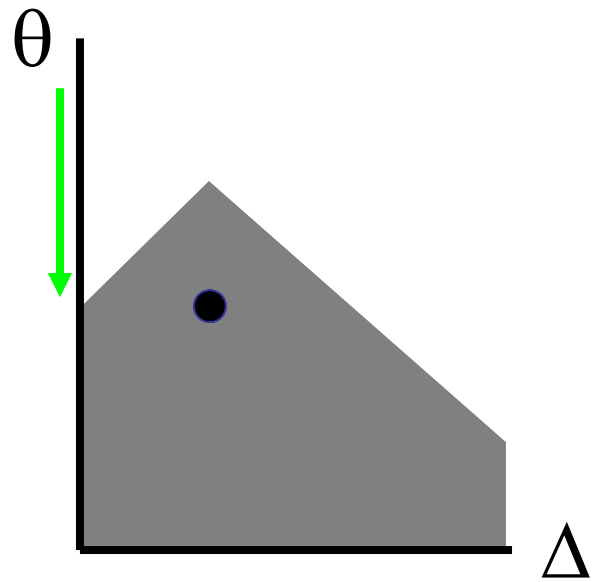
Integrator in action

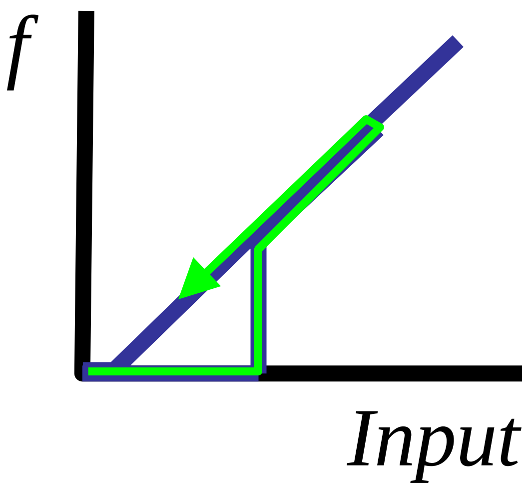


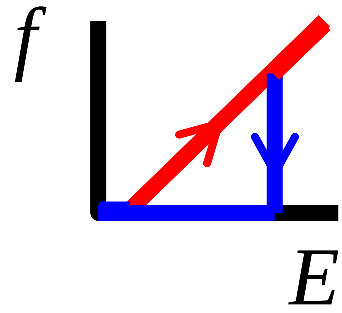
Drop in recurrent current



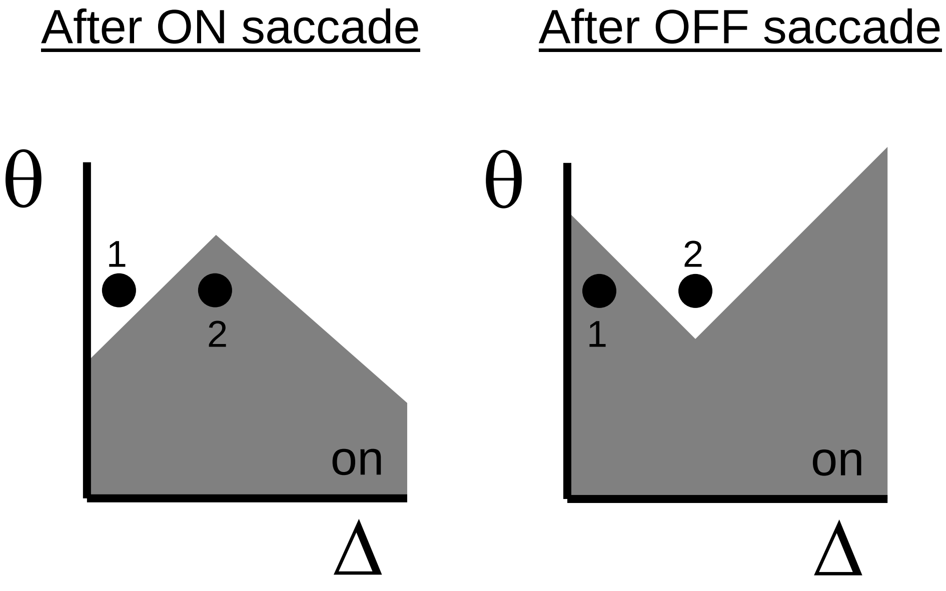
The sign of hysteresis is reversed





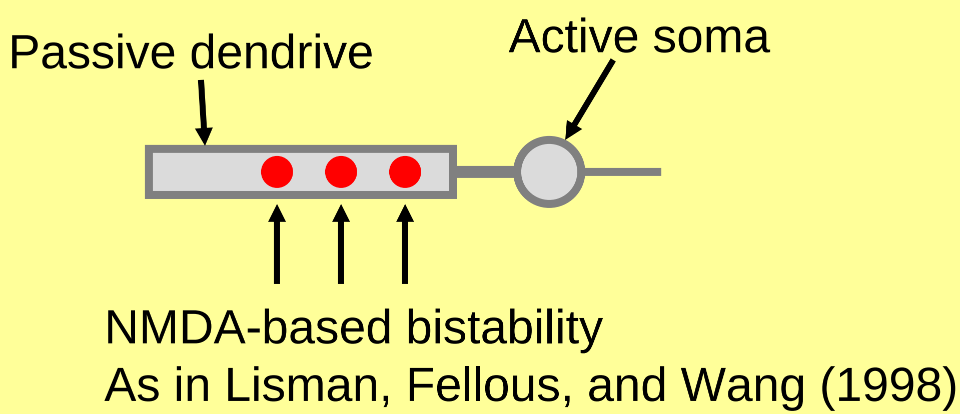


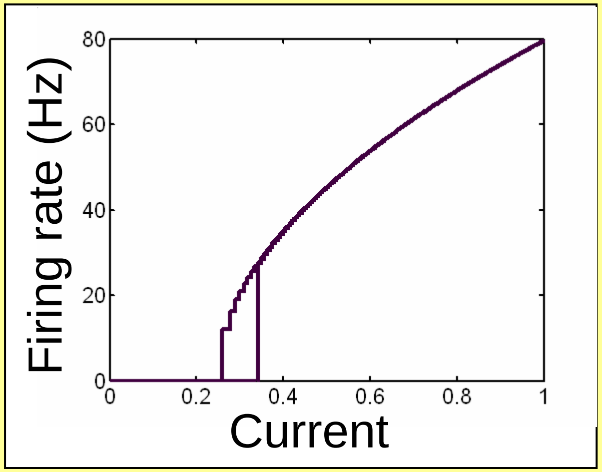
Firing of one neuron versus the other is history-dependent

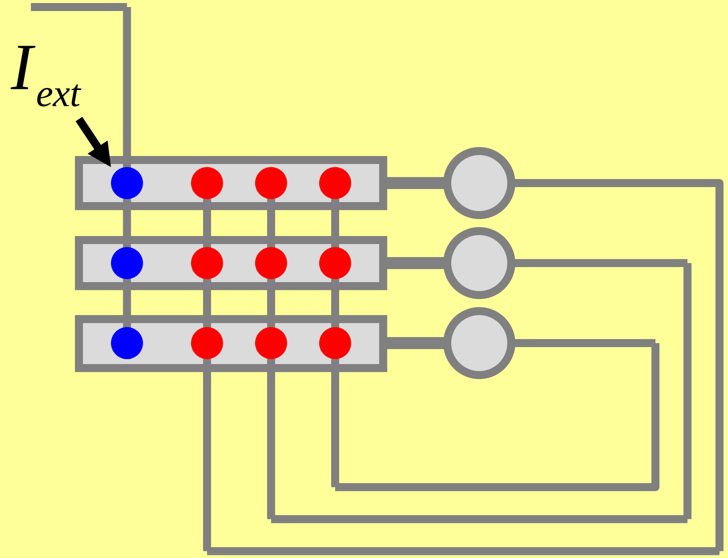


2) A more realistic model

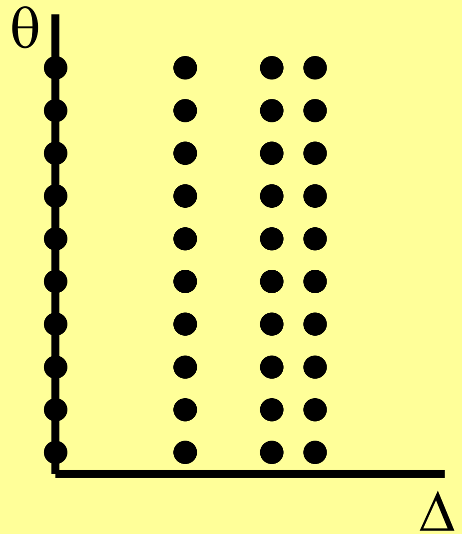
Realistic implementation: two-compartmental model neurons with voltage-based dendritic bistability



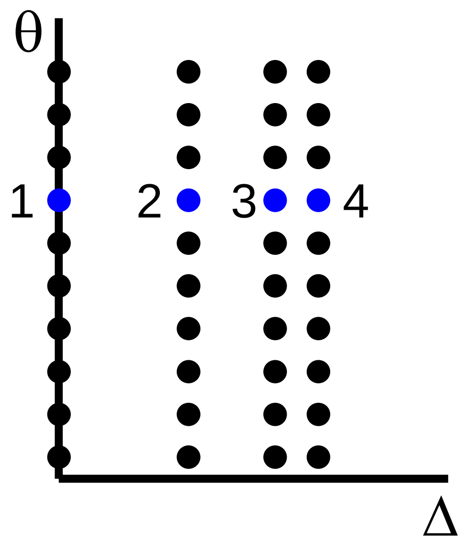


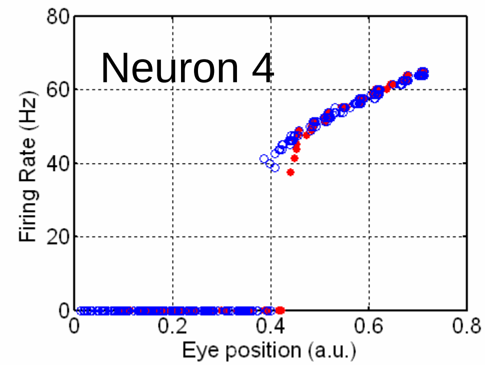


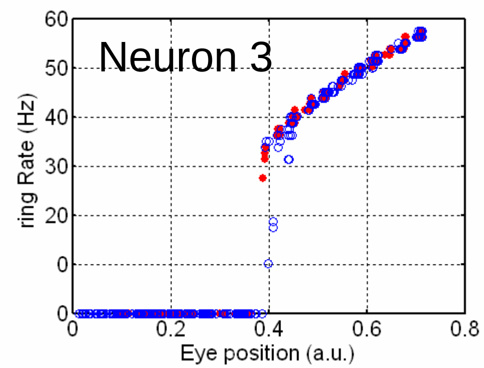
40 neurons:

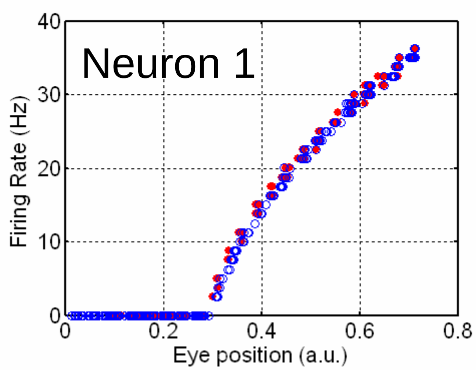


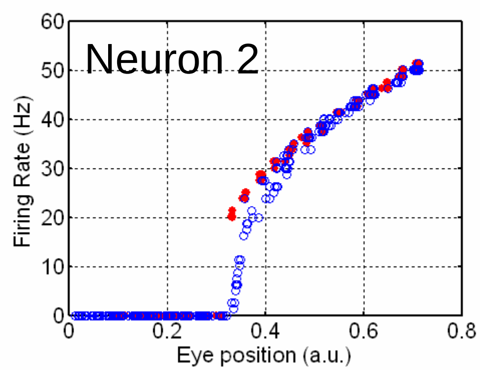
Hysteresis as a function of eye position is reversed in some units







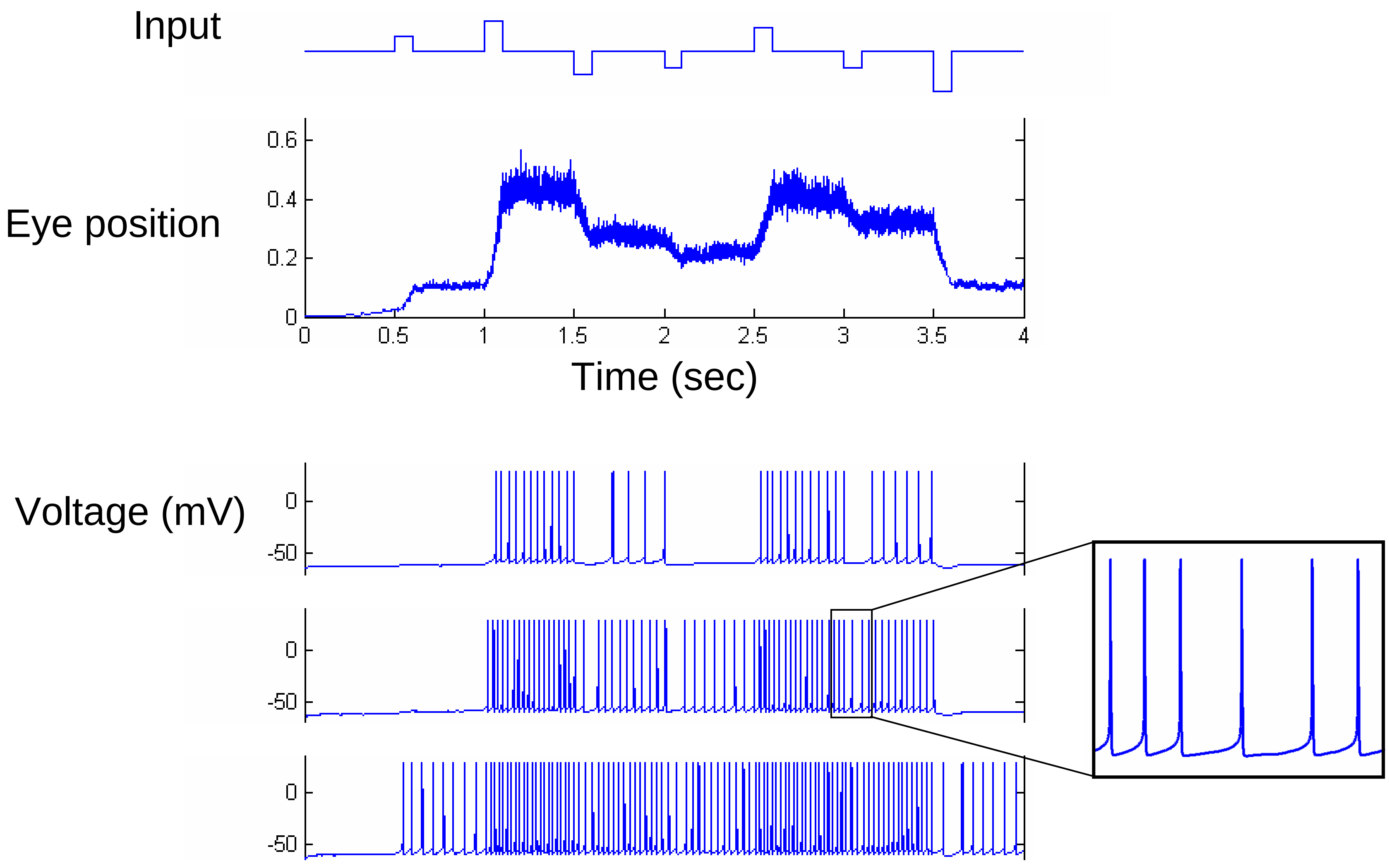




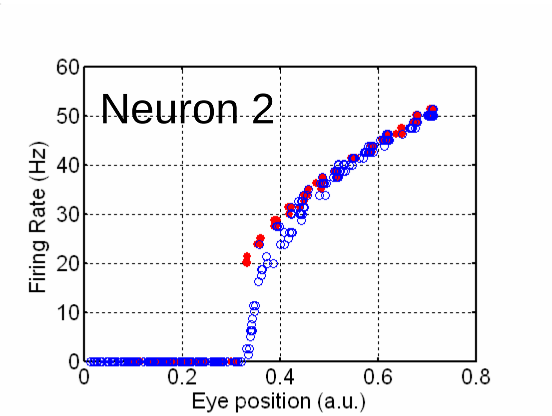
● ON fixations

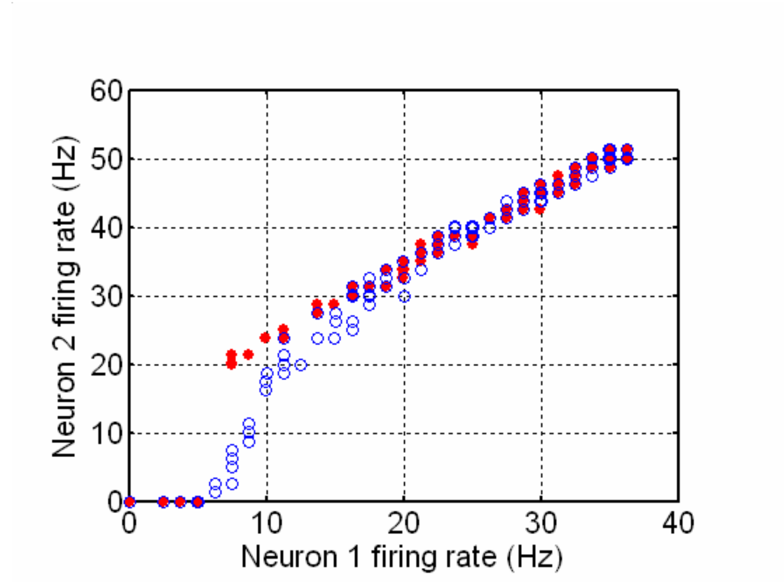
○ OFF fixations

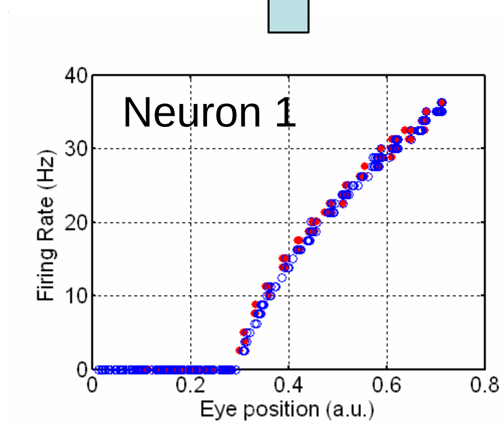
Ensemble of hysteretic units acts as an integrator



History dependence in the firing rate of one neuron versus the other is observed

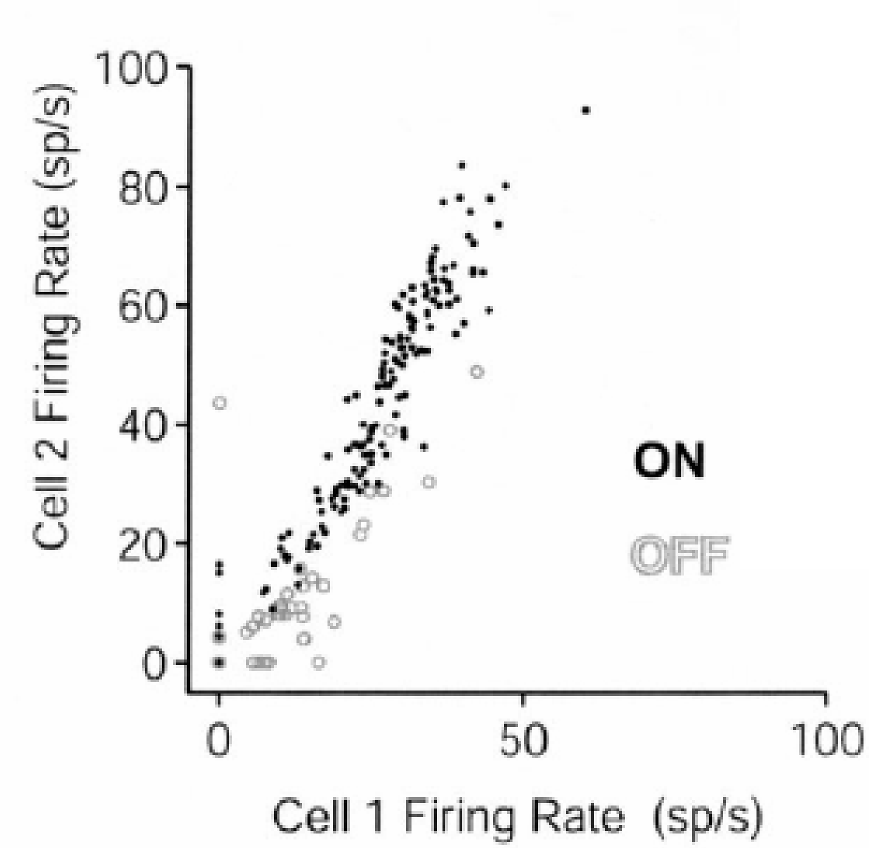






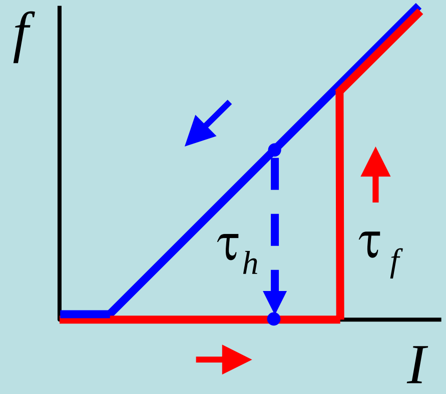
- ● ON fixations
- ○ OFF fixations

Hysteresis in firing of one cell versus the other



Aksay, Major, Goldman, Baker, Seung, Tank, *Cerebral Cortex*, 2003

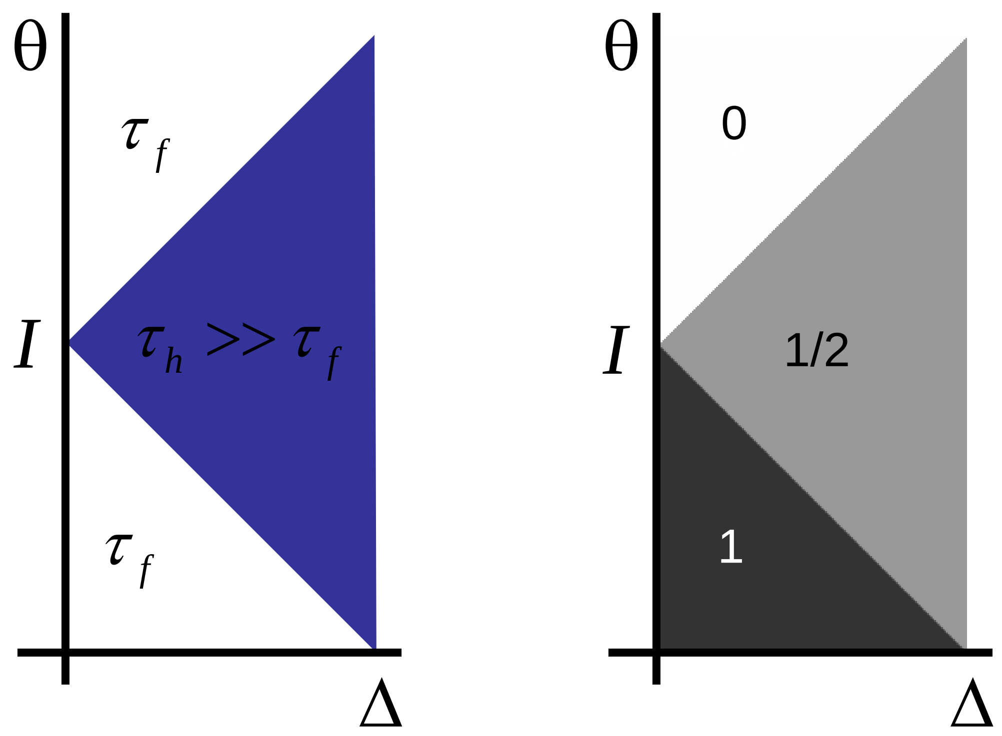
3) Spontaneous transitions



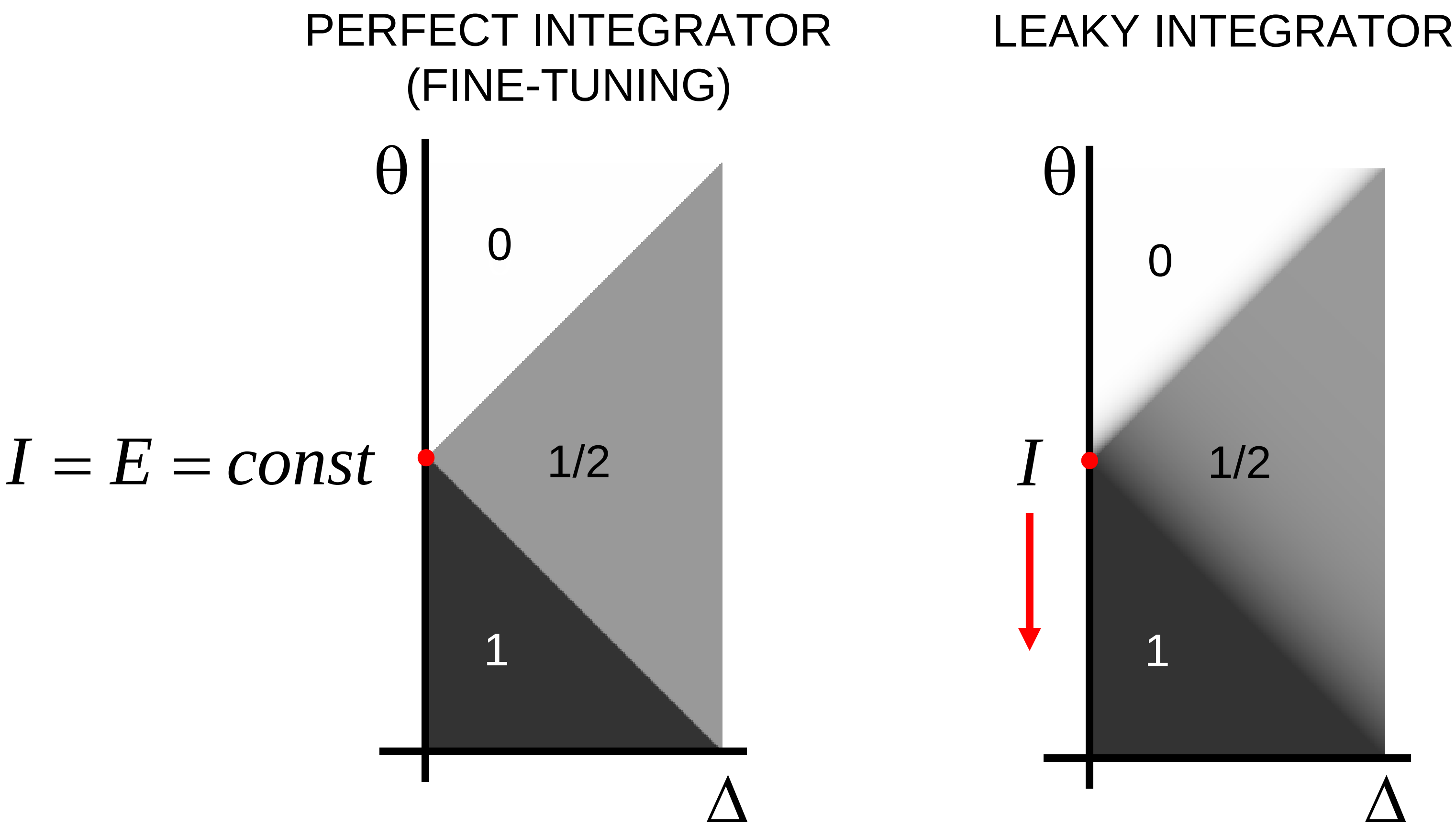
Kinetic equation (KE)

$$\tau(\Delta,\theta,I)\,\dot{f}(\Delta,\theta) = f_0(\Delta,\theta,I) - f(\Delta,\theta)$$

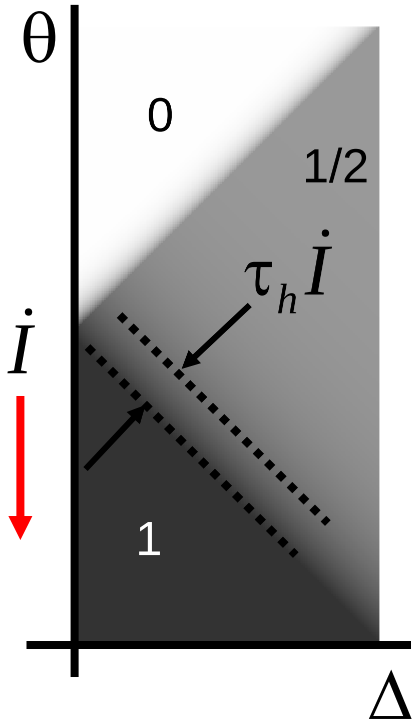
$$\tau(\Delta,\theta,I) \qquad\qquad f_0(\Delta,\theta,I)$$



Solutions of the KE



Leak depends on the rate of spontaneous transitions



$$I_r = I_0 n = C\left(I_r + \tau_h\left(-\dot{I}\right)\right)$$

$$C\tau_h \dot{I} = \left(C-1\right) I$$

$$\tau \approx \tau_h / \left|C-1\right|$$

Conclusion

Different hysteresis -> history dependence in firing

$$\tau = \tau_f / \varepsilon \quad \Rightarrow \quad \tau = \tau_f \exp\left(f \tau_f \right) / \varepsilon$$