Hysteretic models for a neural integrator

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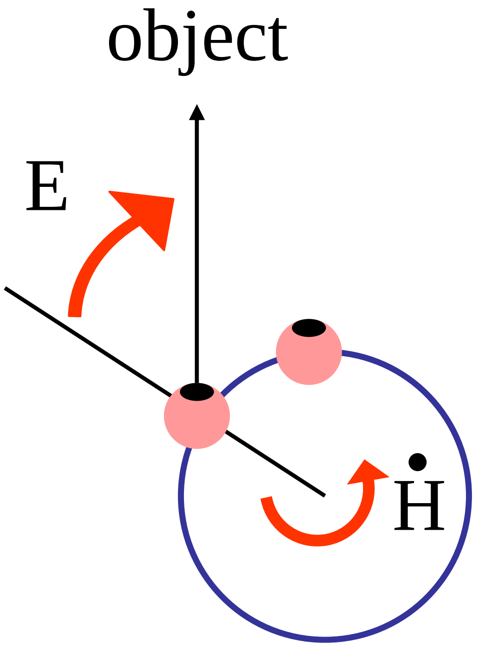
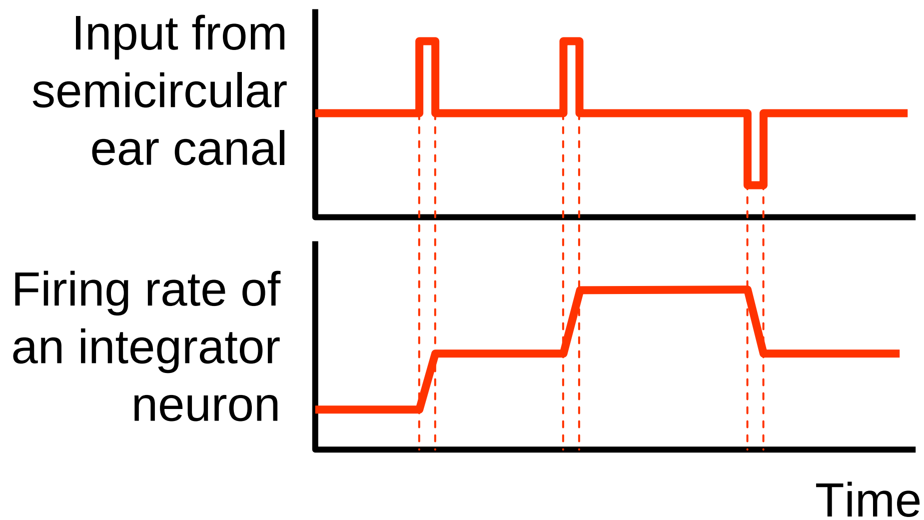
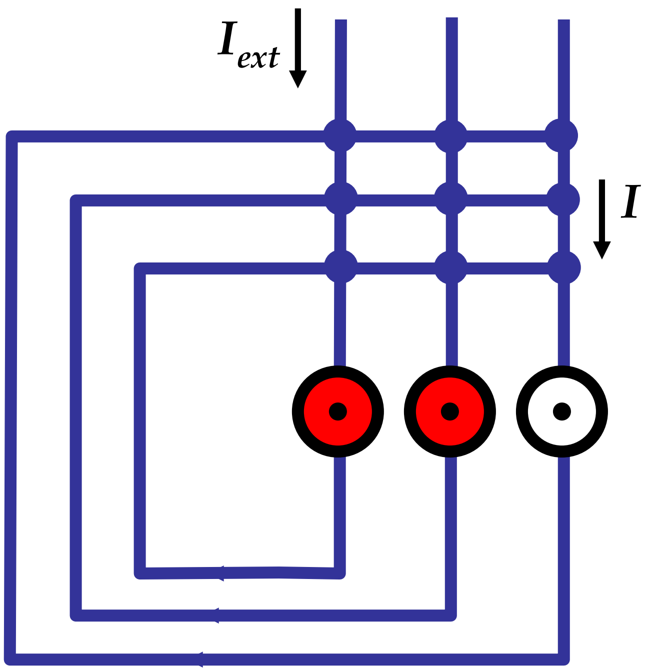
Oculomotor Integrator

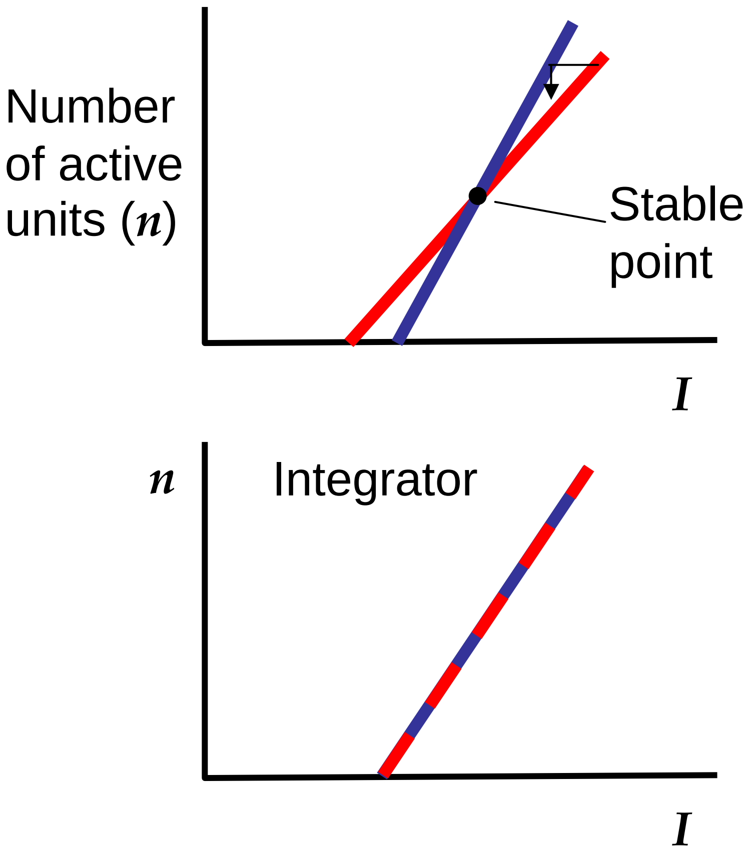


$$E = -\int \dot{H}(t)\,dt$$

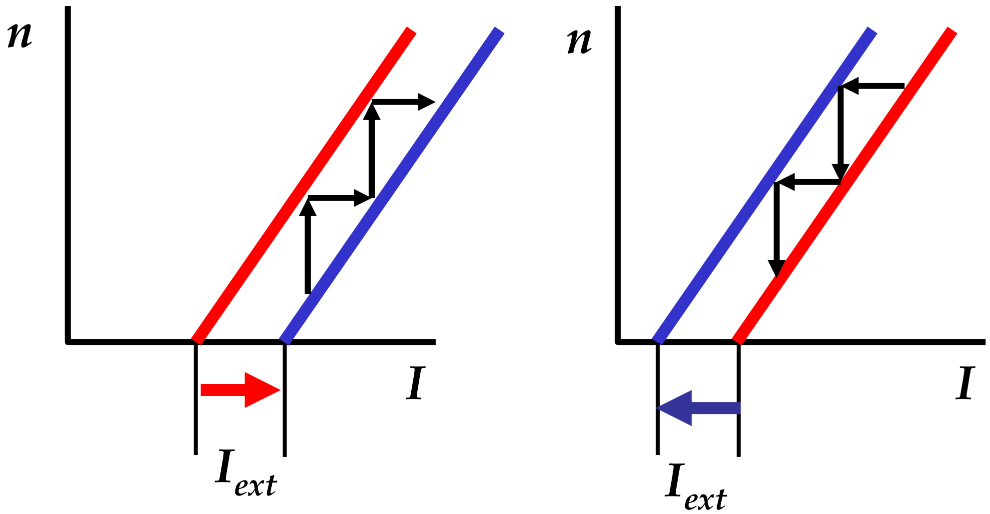


Positive Feedback Mechanism

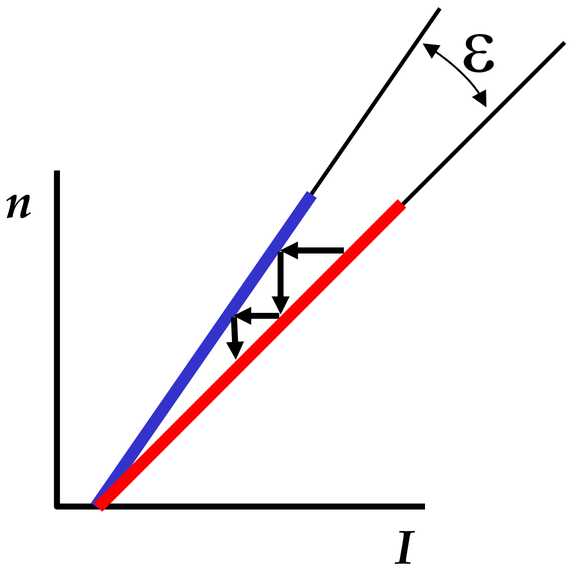




Temporal Integration



Leaky Integrator



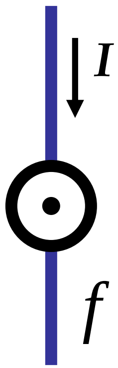
$$\tau = \tau_f N = \tau_f / \varepsilon$$

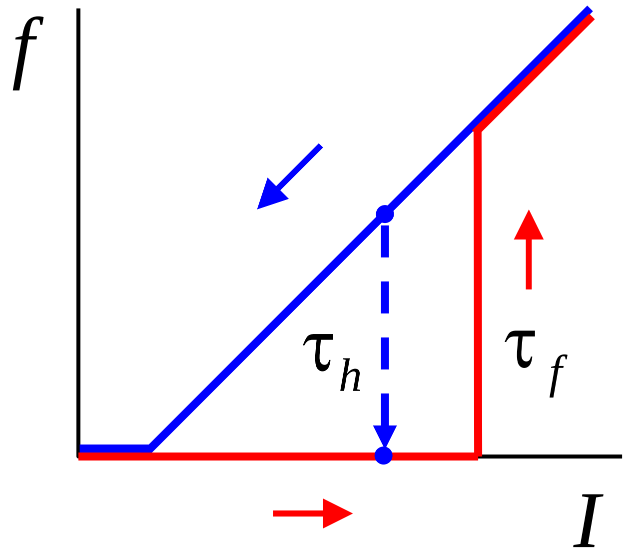
$$\tau_f = 0.1\sec$$

$$\tau = 30\sec$$

$$\varepsilon \sim 0.003$$

Hysteretic units solve the problem of robustness



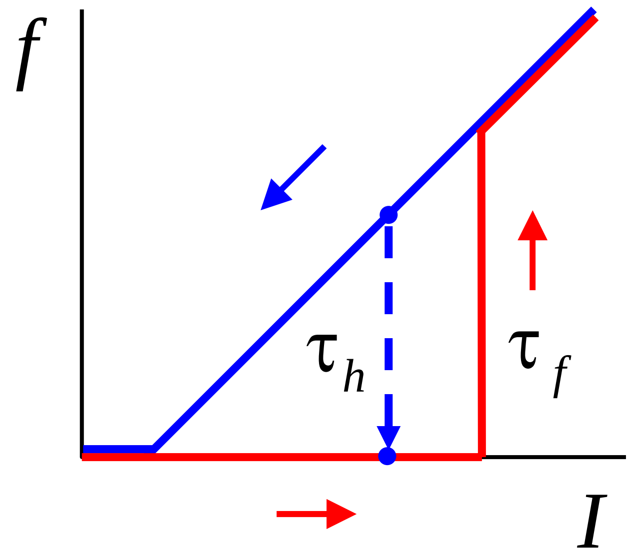


Rosen (1972)
Koulakov, Raghavachari, Kepecs, and Lisman (2002)

Leak time $\tau = \tau_h / \varepsilon$

$\tau_f \ll \tau_h$

Hysteretic units solve the problem of robustness



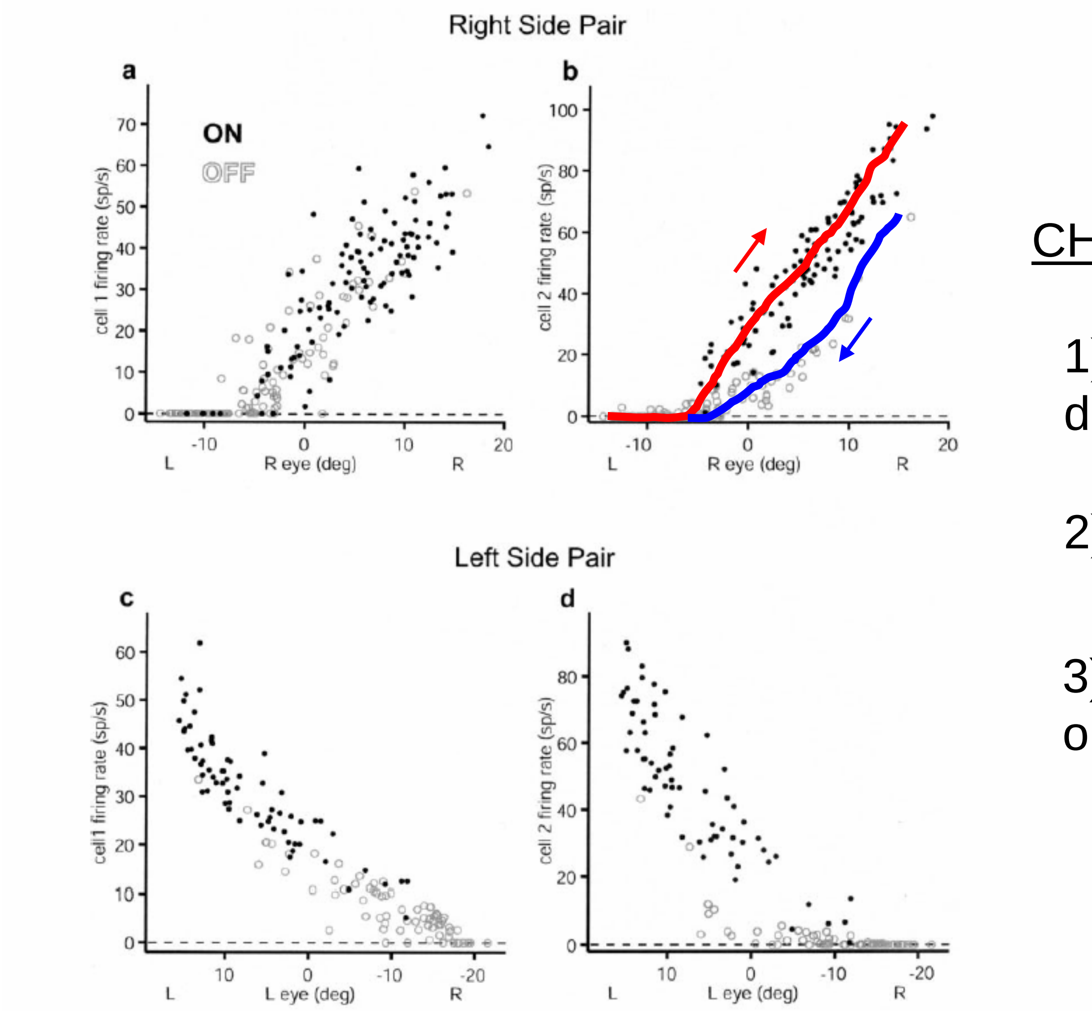
Integration: $\tau_f \sim 0.1\mathrm{sec}$

Memory (leak time): $\tau = \tau_h / \varepsilon$

$$\tau_h = \tau_f \exp\left[C\left(f\tau_f \right) N_{syn} \left(\Delta I / I \right)^3 \right] >> \tau_f$$

Koulakov, 1999

Hysteresis is observed in during fixations in goldfish area I



CHALLENGES FOR THEORY:

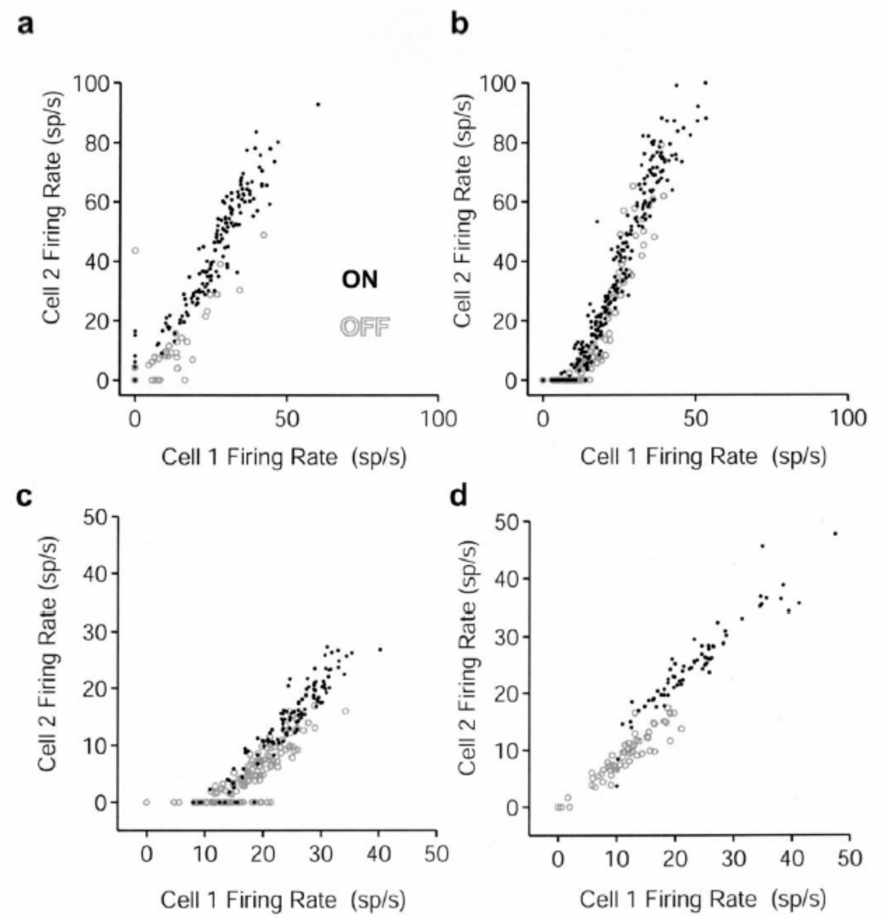
1) Hysteresis is predicted during VOR (not fixations).

2) Wrong sign of hysteresis

3) Hysteresis in firing of one cell versus the other

Aksay, Major, Goldman, Baker, Seung, Tank, *Cerebral Cortex*, 2003

3) Hysteresis in firing of one cell versus the other

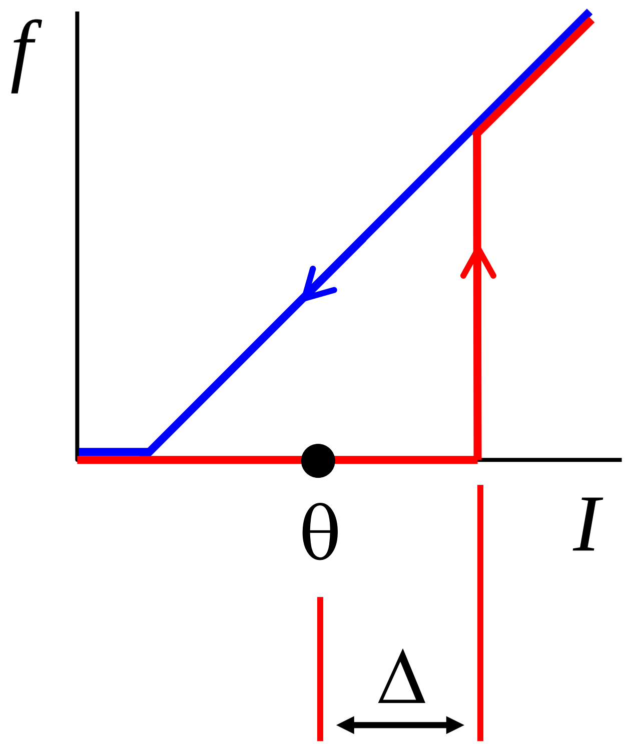


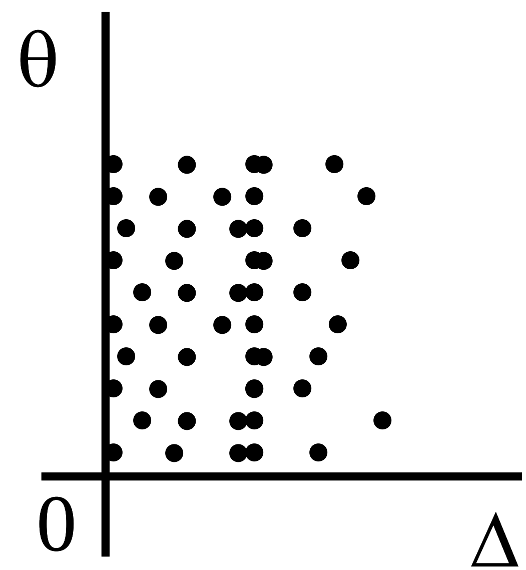
Aksay, Major, Goldman, Baker, Seung, Tank, *Cerebral Cortex*, 2003

1) Model with differential hysteresis

Model based on inhomogeneous ensemble of hysteretic units

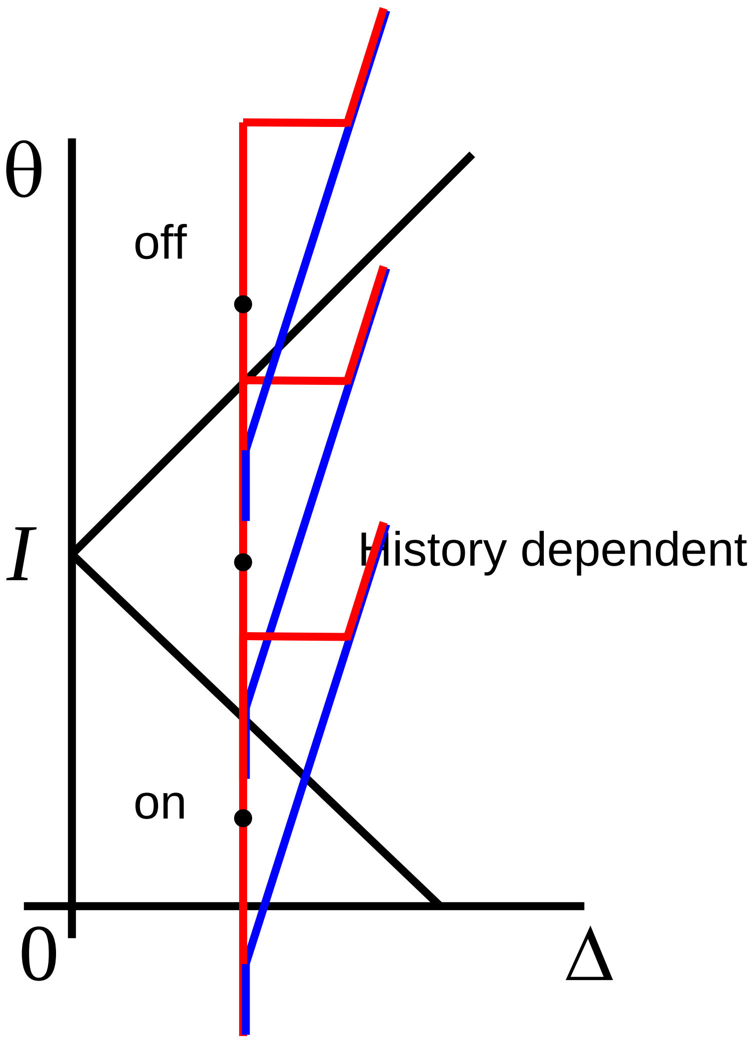
Before this model
This (Koulakov et. al., 2002)



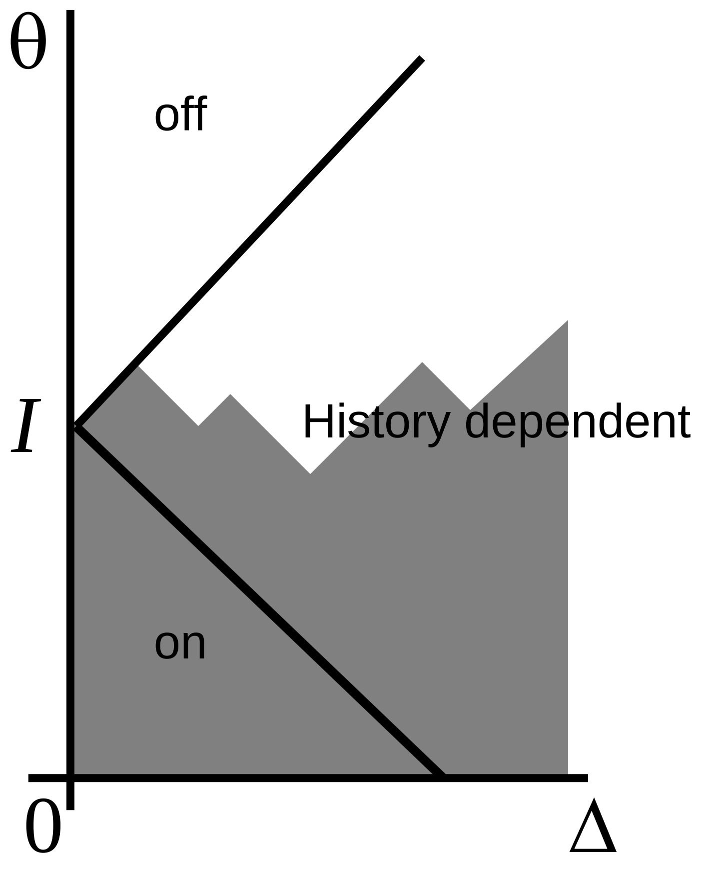


$$\rho(\Delta) = \rho_0 \exp\left(-\Delta / \overline{\Delta}\right)$$

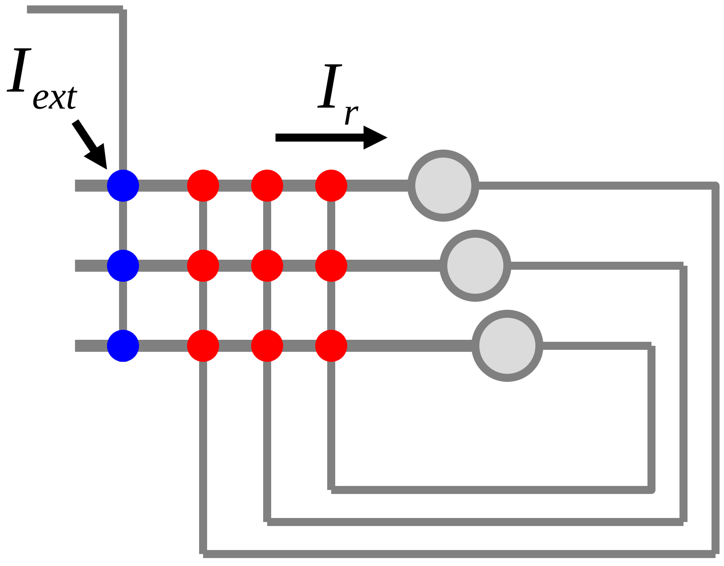
The distribution function of active units is history- dependent



Possible activation profile

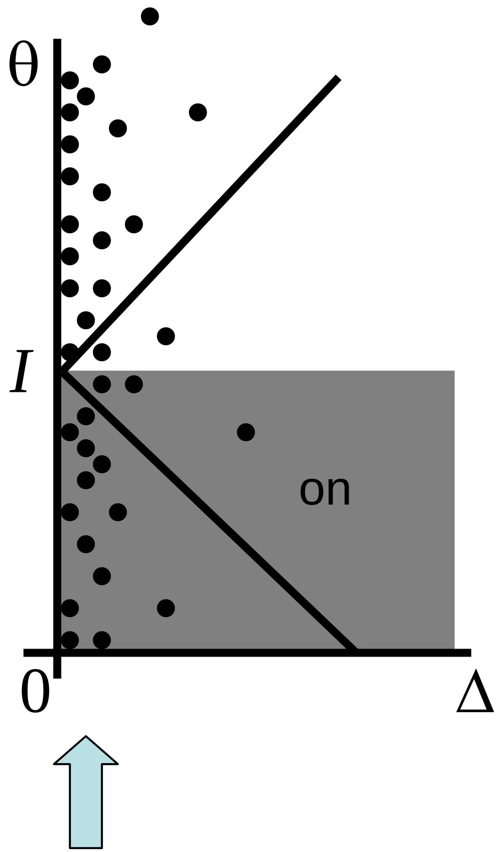


Model with feedback can act as integrator



$$I = I_r + I_{ext}$$

Additional constraint: Stability condition



$$\rho(\Delta) = \rho_0 \exp\left(-\Delta / \overline{\Delta}\right)$$

<u>During fixations:</u>

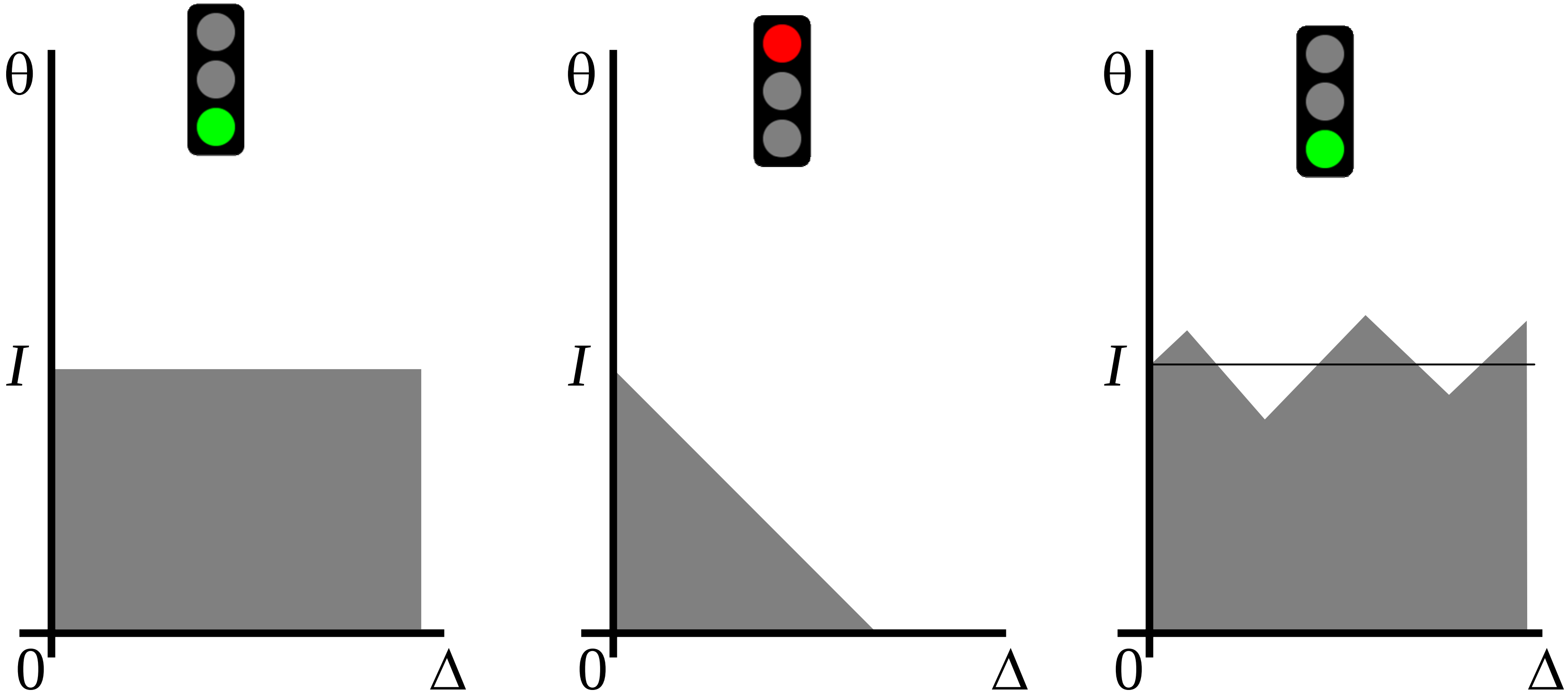
$$I_{ext} = 0$$

$$I = I_r = I_0 n_{on} = I_0 \int f(\Delta, \theta) \rho(\Delta) d\Delta d\theta =$$

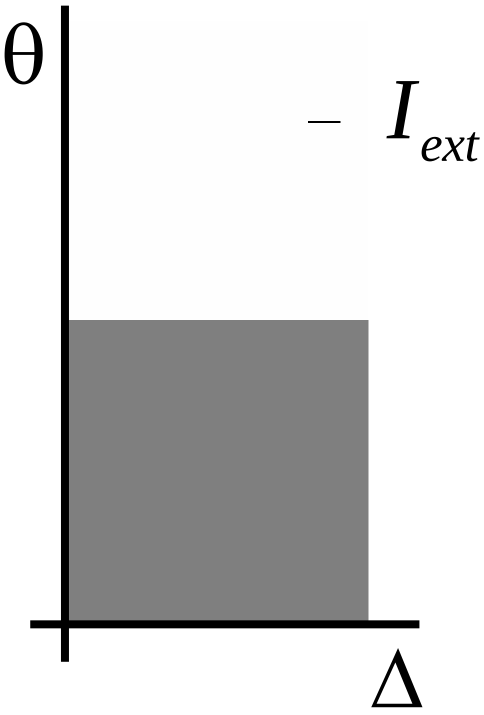
$$= I_0 \rho_0 \overline{\Delta} \cdot I = CI$$

$$f(\Delta, \theta) = \begin{cases} 1, \text{ for ON units} \\ 0, \text{ for OFF units} \end{cases}$$

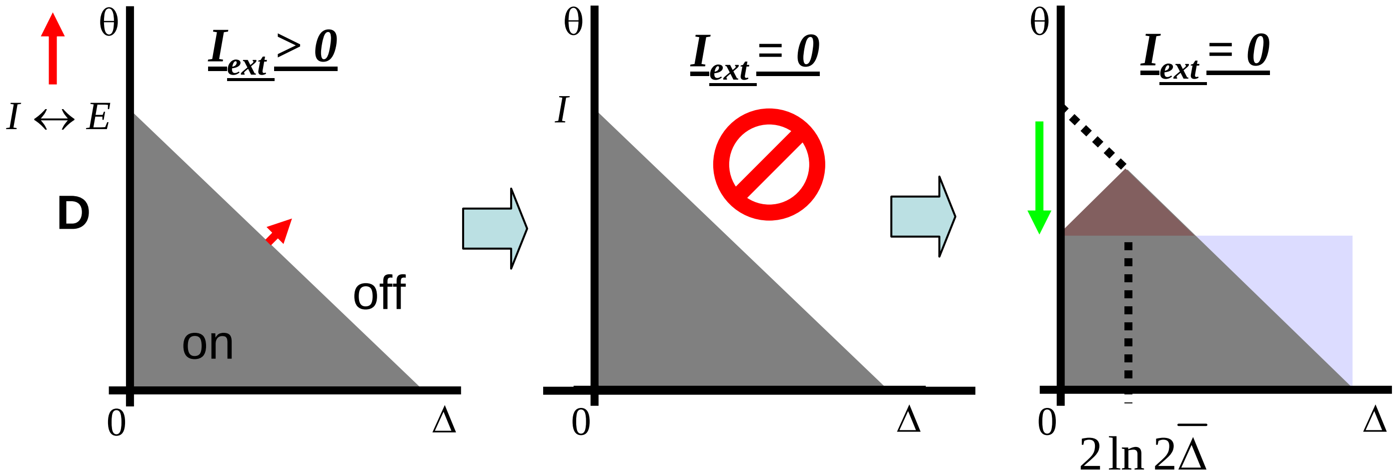
Possible solutions



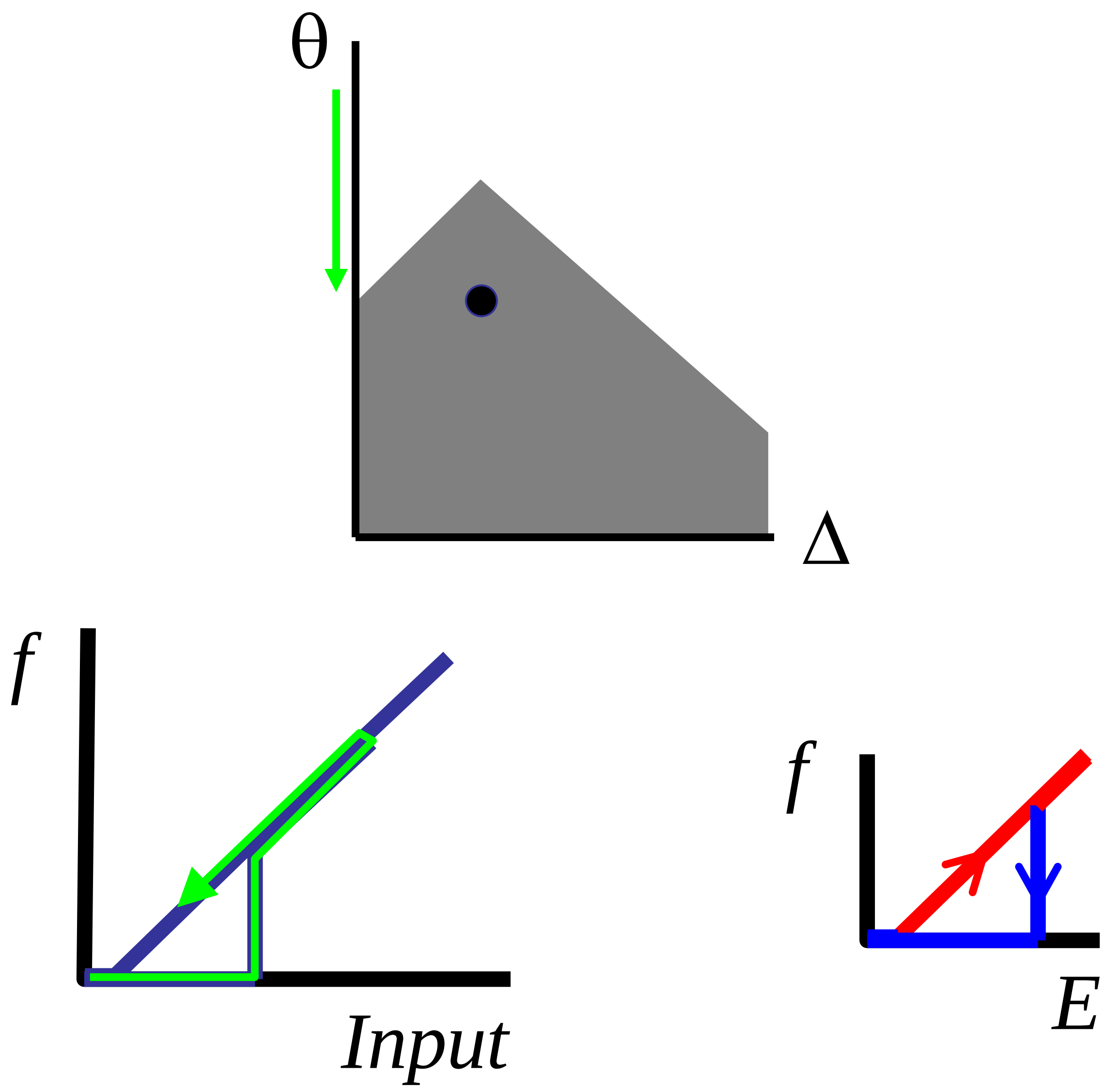
Integrator in action



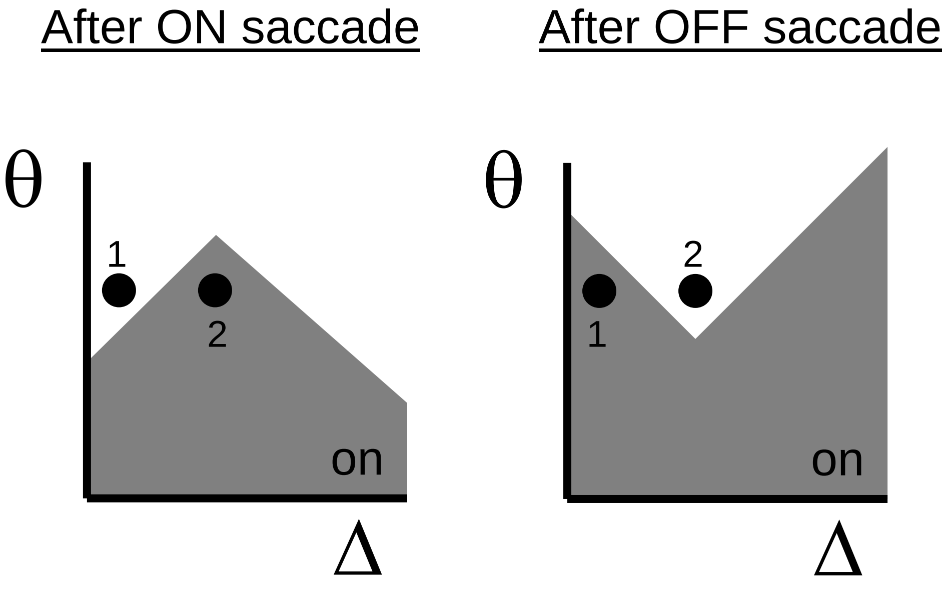
Drop in recurrent current



The sign of hysteresis is reversed

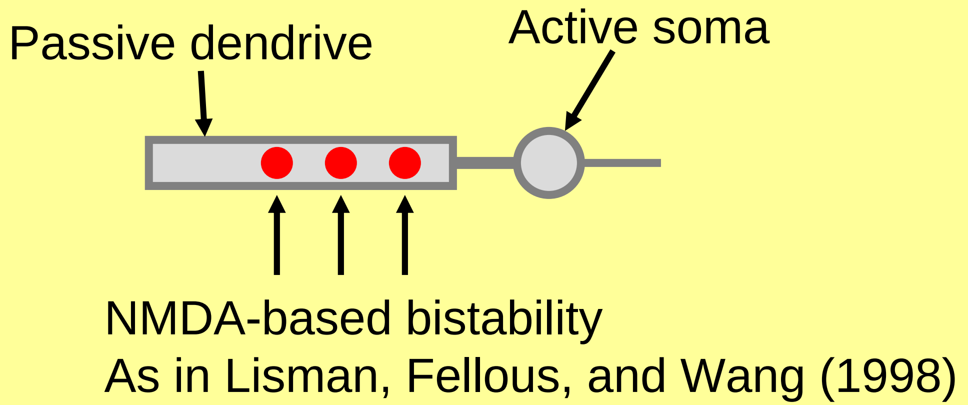


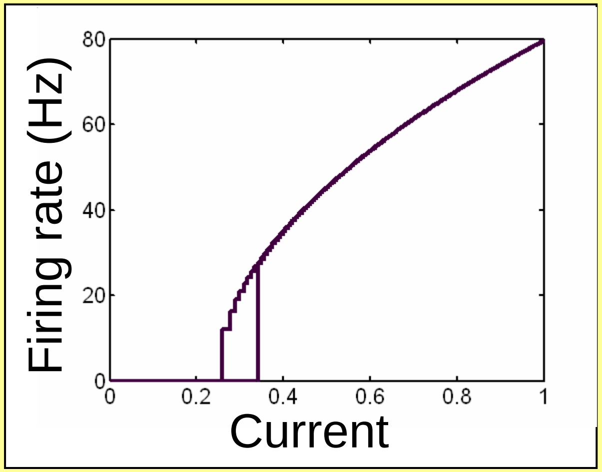
Firing of one neuron versus the other is history-dependent

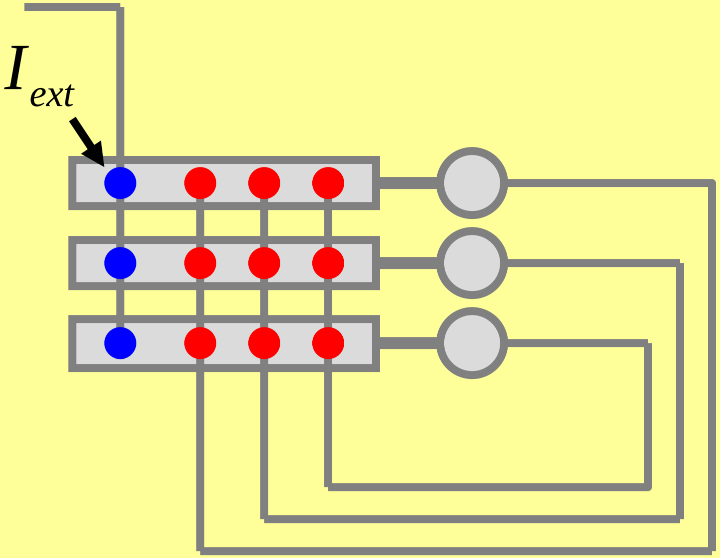


2) A more realistic model

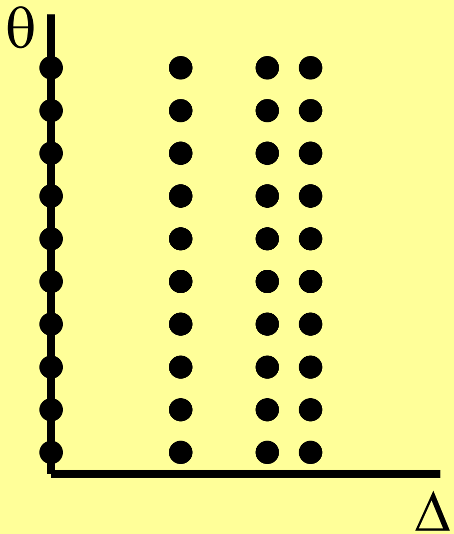
Realistic implementation: two-compartmental model neurons with voltage-based dendritic bistability



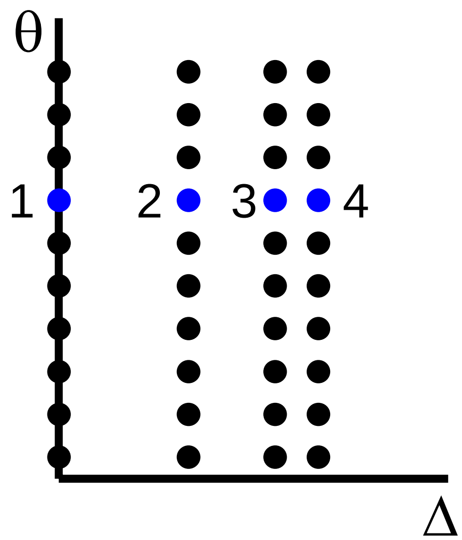


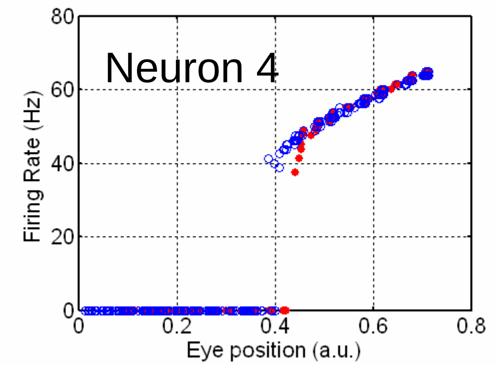


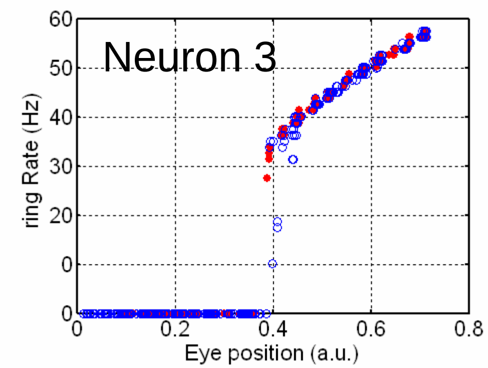
40 neurons:

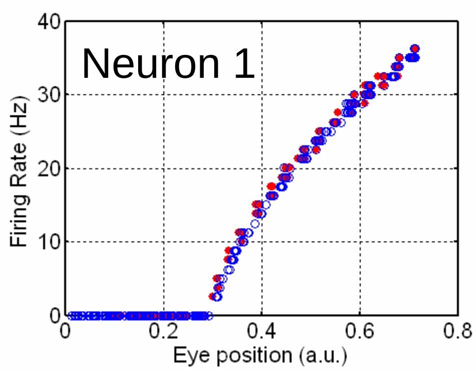


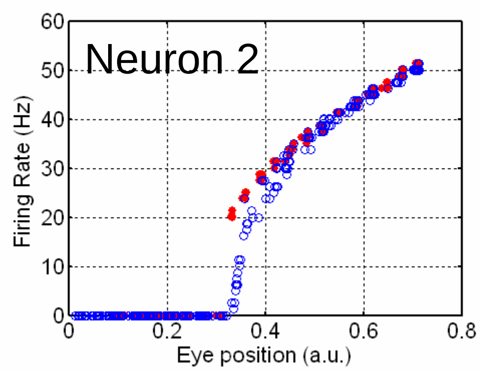
Hysteresis as a function of eye position is reversed in some units







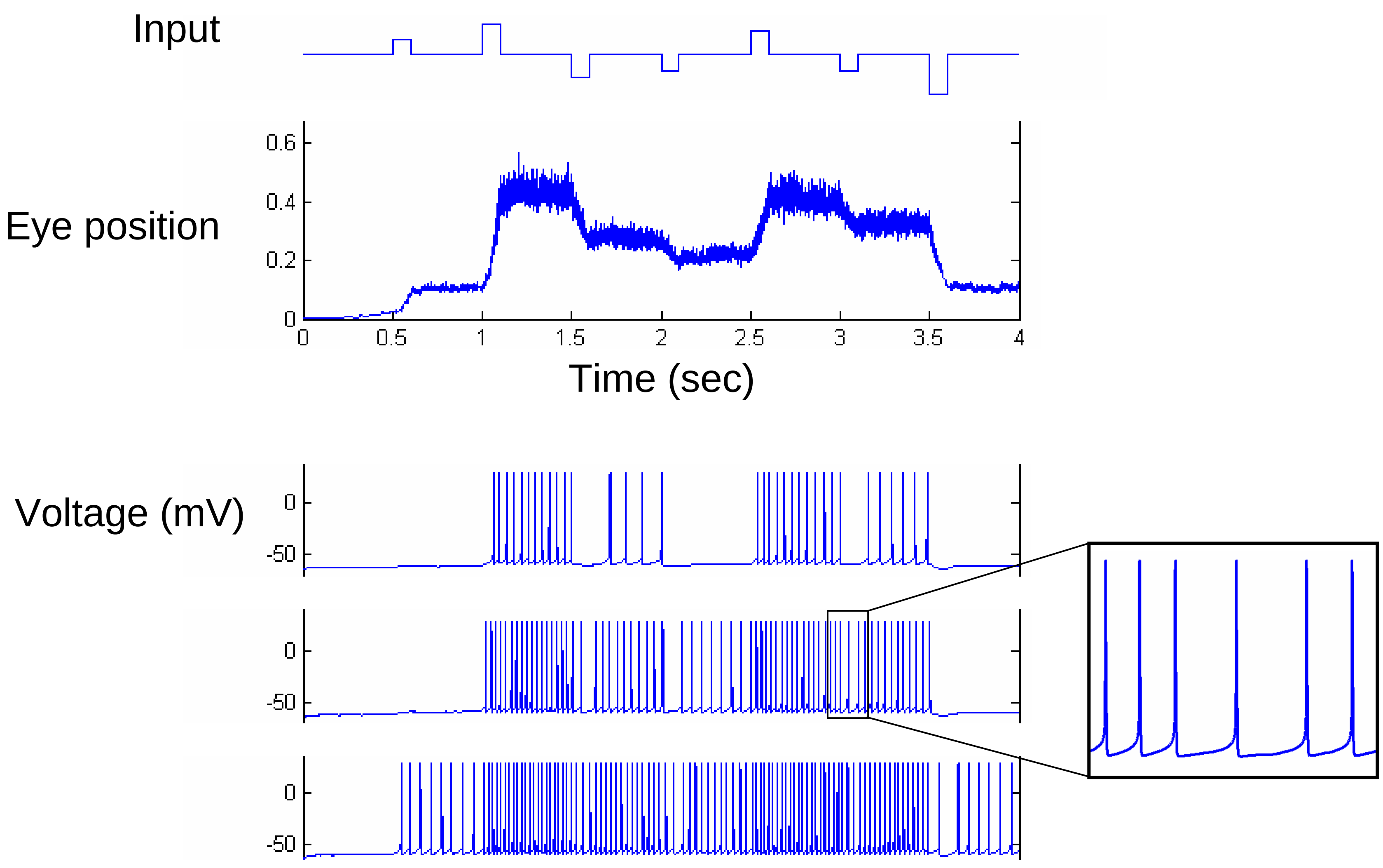




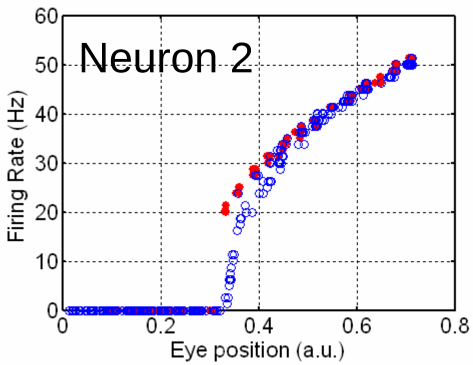
● ON fixations

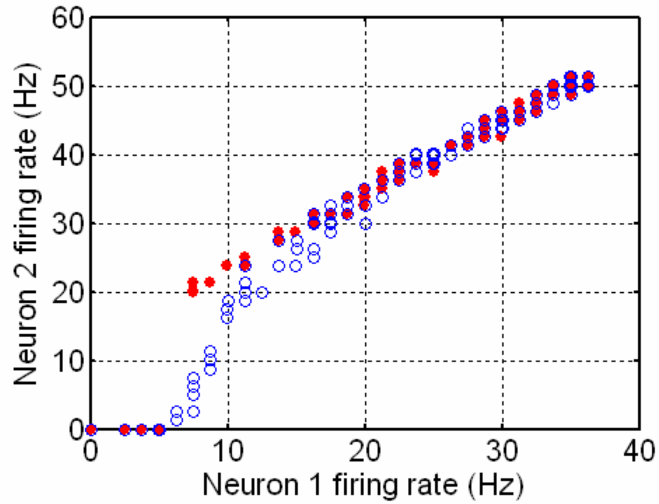
○ OFF fixations

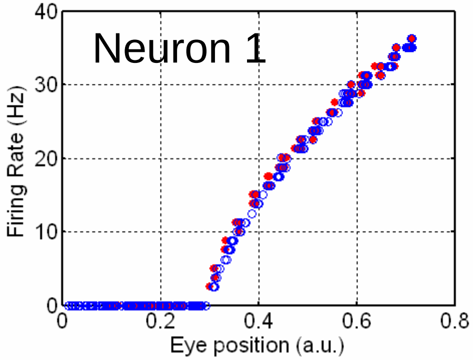
Ensemble of hysteretic units acts as an integrator



History dependence in the firing rate of one neuron versus the other is observed

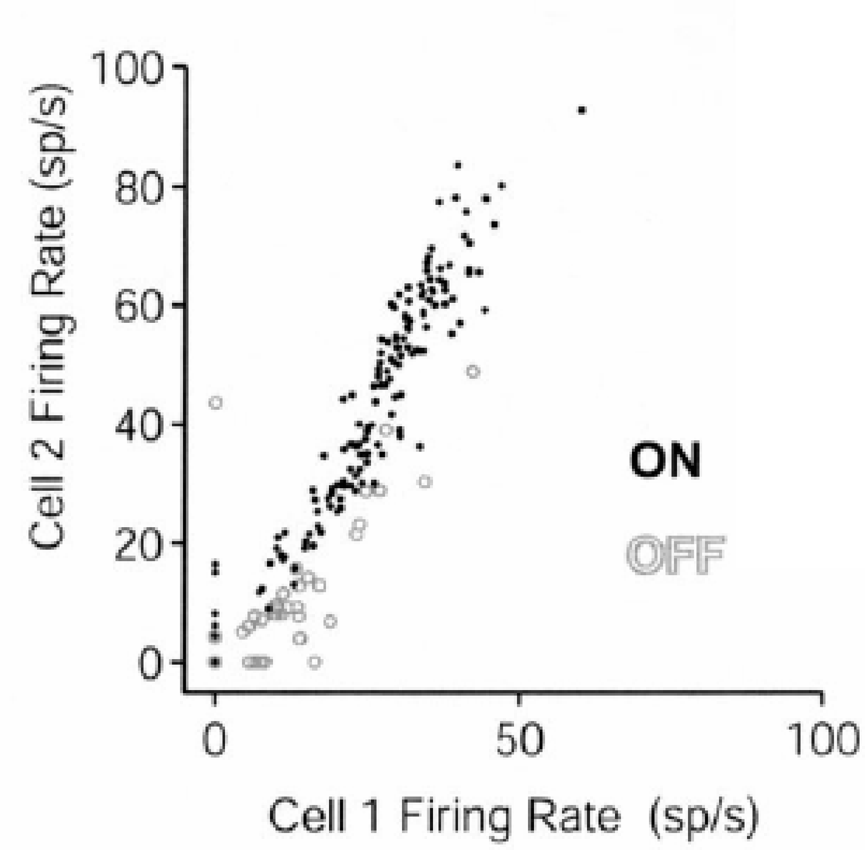






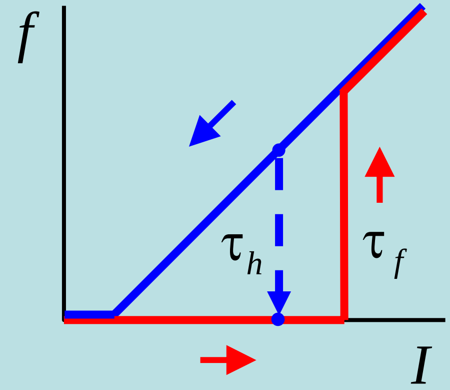
- ● ON fixations
- ○ OFF fixations

Hysteresis in firing of one cell versus the other



Aksay, Major, Goldman, Baker, Seung, Tank, *Cerebral Cortex*, 2003

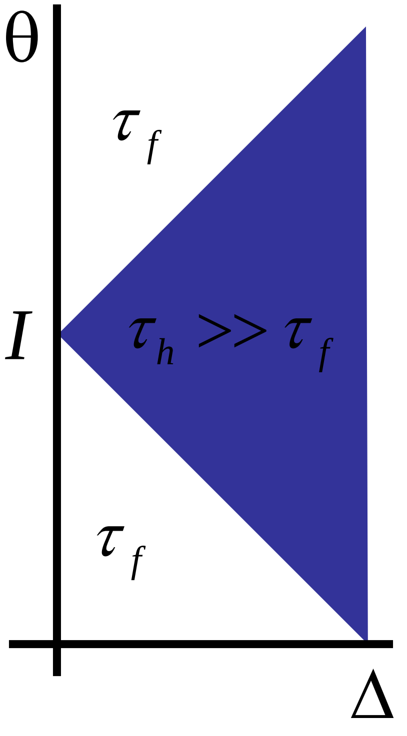
3) Spontaneous transitions



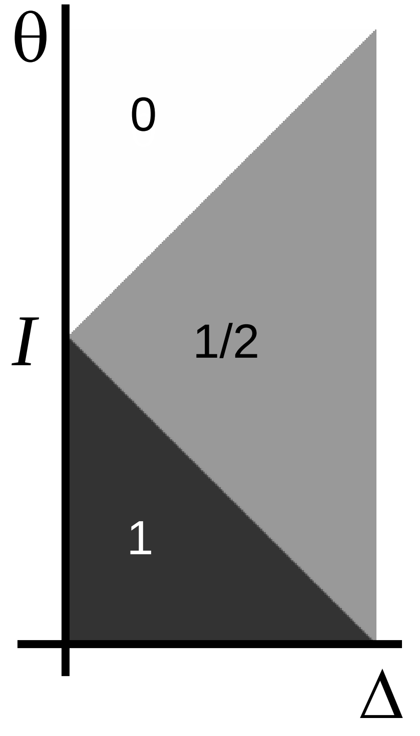
Kinetic equation (KE)

$$\tau(\Delta,\theta,I)\,\dot{f}(\Delta,\theta) = f_0(\Delta,\theta,I) - f(\Delta,\theta)$$

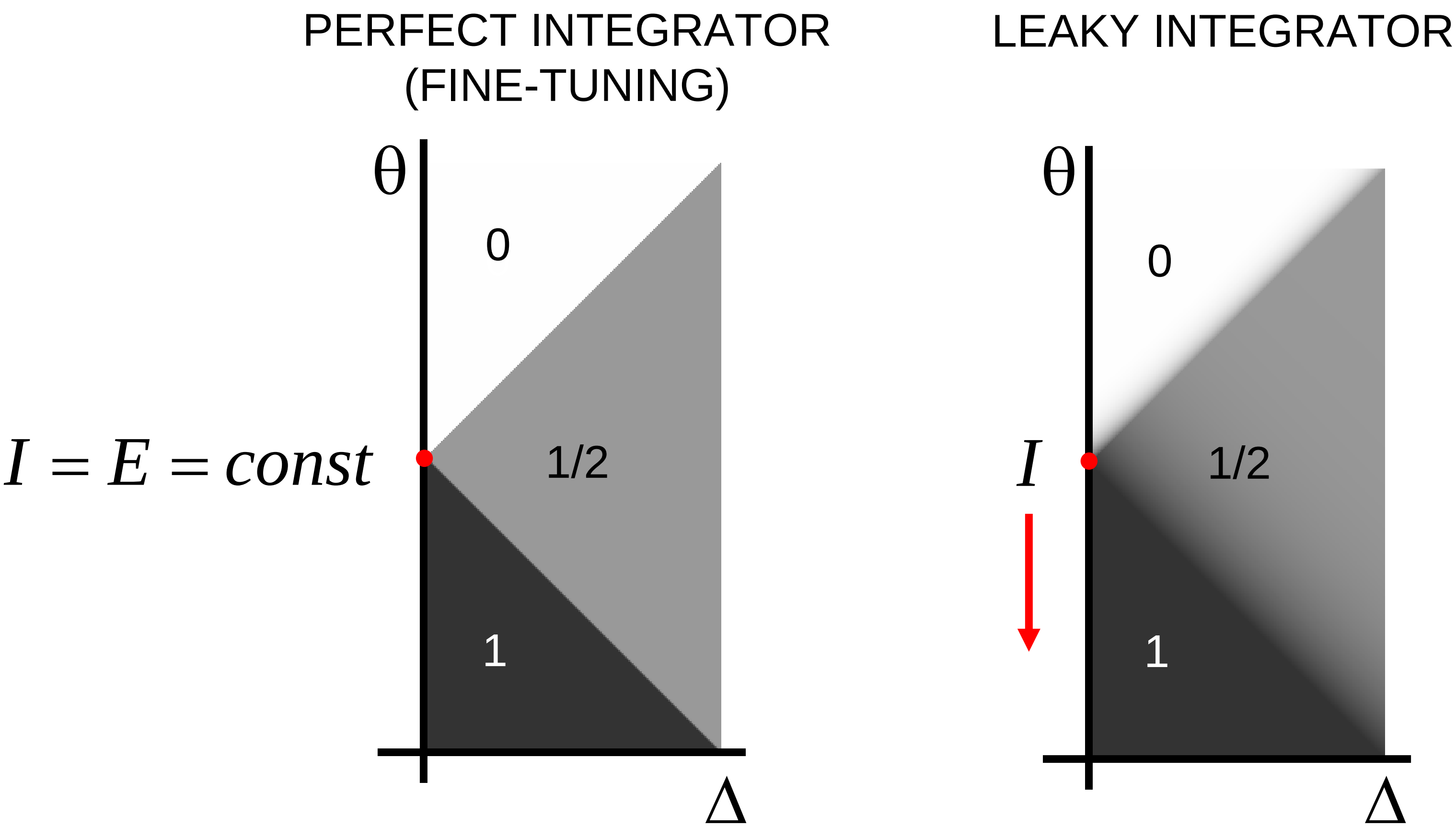
$\tau(\Delta,\theta,I)$



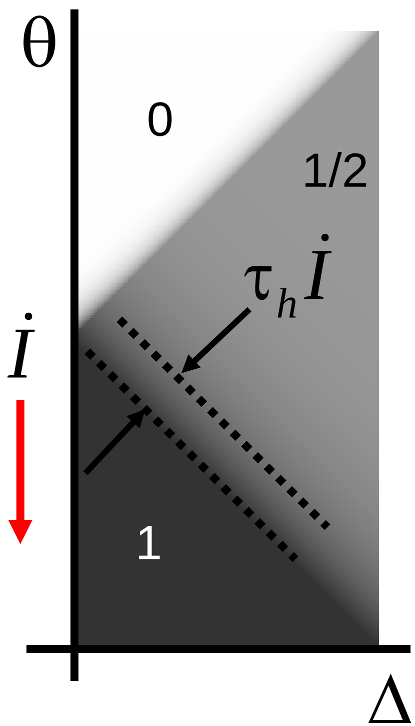
$f_0(\Delta,\theta,I)$



Solutions of the KE



Leak depends on the rate of spontaneous transitions



$$I_r = I_0 n = C\left(I_r + \tau_h\left(-\dot{I}\right)\right)$$

$$C\tau_h \dot{I} = \left(C-1\right) I$$

$$\tau \approx \tau_h / \left|C-1\right|$$

Conclusion

Different hysteresis -> history dependence in firing

$$\tau = \tau_f / \varepsilon \quad \Rightarrow \quad \tau = \tau_f \exp\left(f \tau_f \right) / \varepsilon$$