Correlations and Synchrony in Threshold Neuron Models

Presentation by Tatjana Tchumatchenko

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Acknowledgements

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Why study spike correlations?

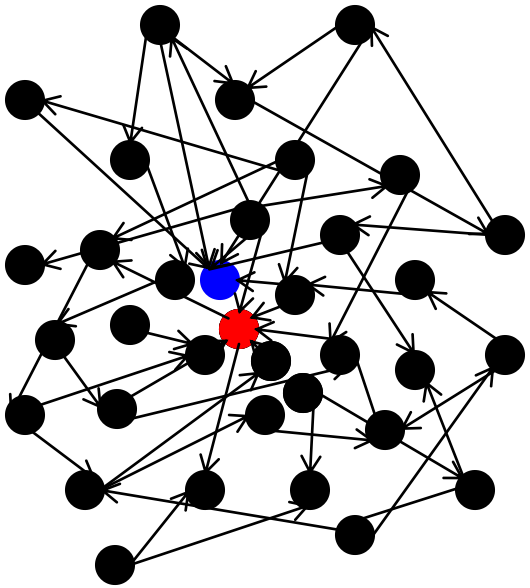
Correlated neural activity

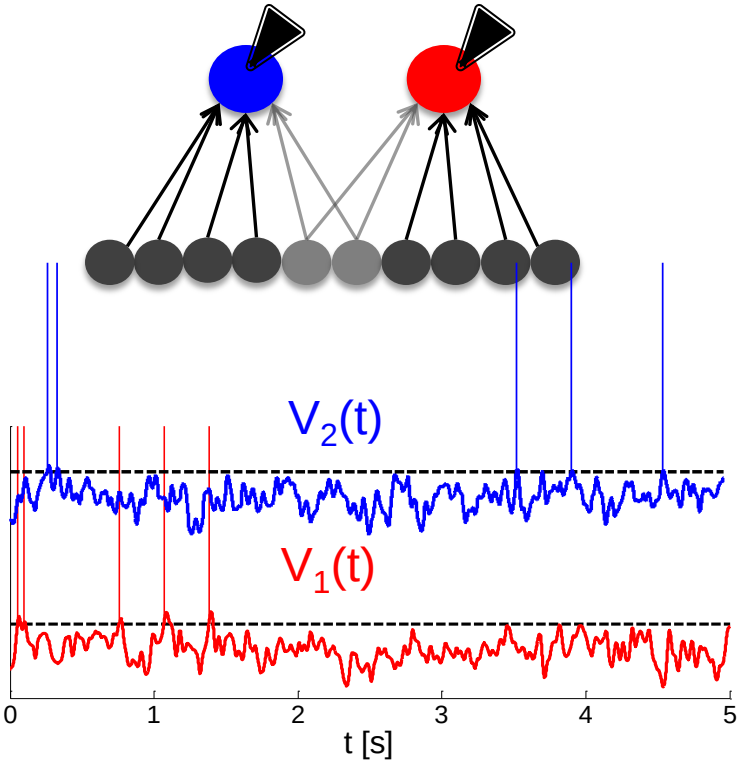
- Influences population encoding and decoding[1,2]
- Is a basis for Hebbian learning[3]
- Can reflect specific features of sensory stimuli or behavioral tasks[4,5]

1 L.F. Abbott and P. Dayan., *Neural Comput*, 11(1):91–101, (1999)
2 M. Shamir and H. Sompolinsky, *Neural Comput*., 16:1105–1136, (2004)
3 P. Dayan and L. Abbott, MIT press, 2001
4 Panzeri, S. *et al., Vis. Cogn. 8, 531–547 (2001)*
5 Romo, R. *et al.*, *Neuron,* 38, 649–657 (2003)

The key question

Input correlations $\xrightarrow{?}$ Output spike correlations





Experimental findings

Voltages are temporally correlated and can be assumed Gaussian [1,2]

Pairwise cross correlations are typically weak [3]

Spike correlations can be firing rate dependent [3,4,5]

Spike correlations with or without evidence of direct connections can be asymmetric [6]

Temporal width of spike correlations can be significantly smaller than the width of the correlations of the underlying membrane potentials [6]

1 R. Azouz and C. M. Gray, *J. Neurosci.*, 19:2209–2223, (1999)
2 A. Destexhe et al., *Nat. Rev. Neurosci.*, 4:739–751, (2003)
3 D.S. Greenberg et al. *Nat. Neurosci.*, 11(7):749–751, (2008)
4 G. Svirskis and J. Hounsgaard, *Network*, 14:747–763, (2003)
5 J. de la Rocha et al., *Nature*, 448:802–806, (2007)
6 I. Lampl et al., *Neuron*, 22:361–374, (1999)

Open questions

- What determines pairwise spike correlations?
- How is temporal structure of input currents reflected in the spike correlations?
- What is the correlation structure in a pair with different firing rates?

Results

- What determines pairwise spike correlations?

  Weak correlations: firing rate, correlation strength and correlation time τ_s
  Strong correlations: correlation time and correlation strength

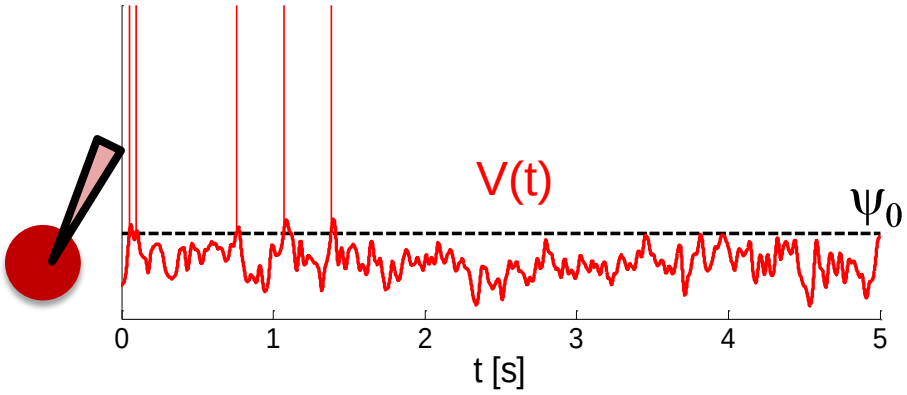
- How is temporal structure of input currents reflected in the spike correlations?

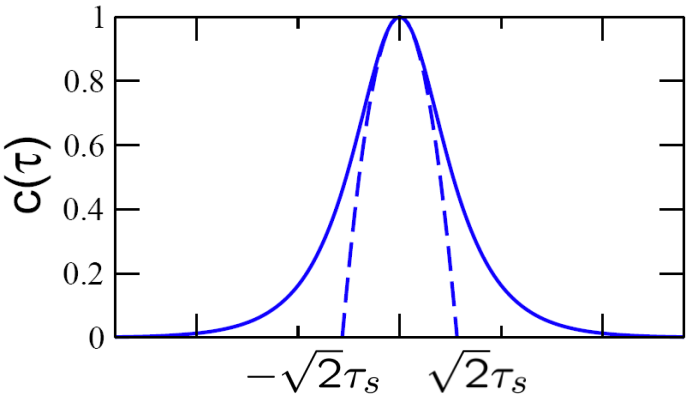
  Reflected only in the weak correlations
  Mixture of voltage correlation time and its second derivative
  Temporal width of spike correlations is firing rate dependent

- What is the correlation structure in a pair with different firing rates?
  Spike correlations are asymmetric

Statistical framework

- V(τ) is a stationary continuous random Gaussian process



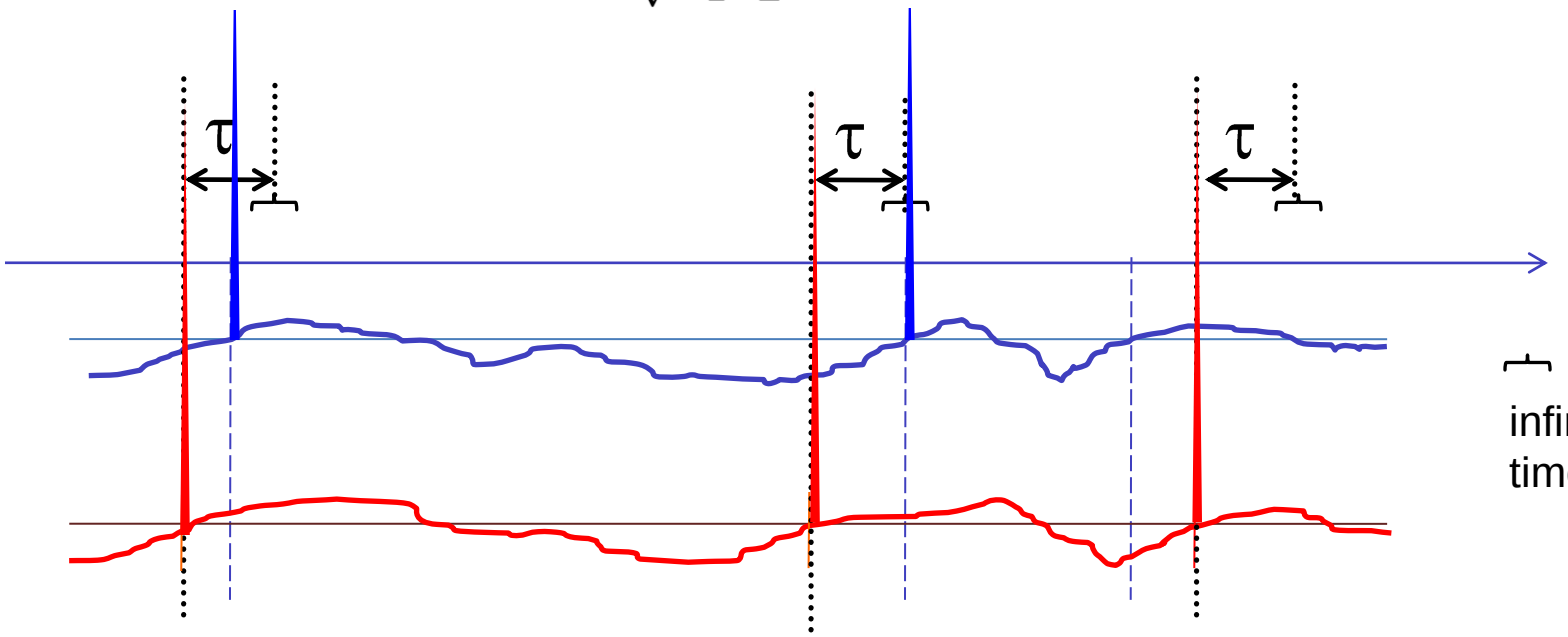


- Spike = positive threshold crossing of V(τ)
- Voltage correlation function: $\langle V(t)V(t+\tau)\rangle = C(\tau) = \sigma_V^2/\cosh(\tau/\tau_s)$
- Stationary firing rate: $\nu = \langle s(t)\rangle = \frac{1}{2\pi\tau_s}\exp\left(-\frac{\psi_0^2}{2\sigma_V^2}\right)$

R. Azouz and C. M. Gray, *J. Neurosci.*, 19:2209–2223, (1999)
A. Destexhe et al., *Nat. Rev. Neurosci.*, 4:739–751, (2003)

Conditional firing rate

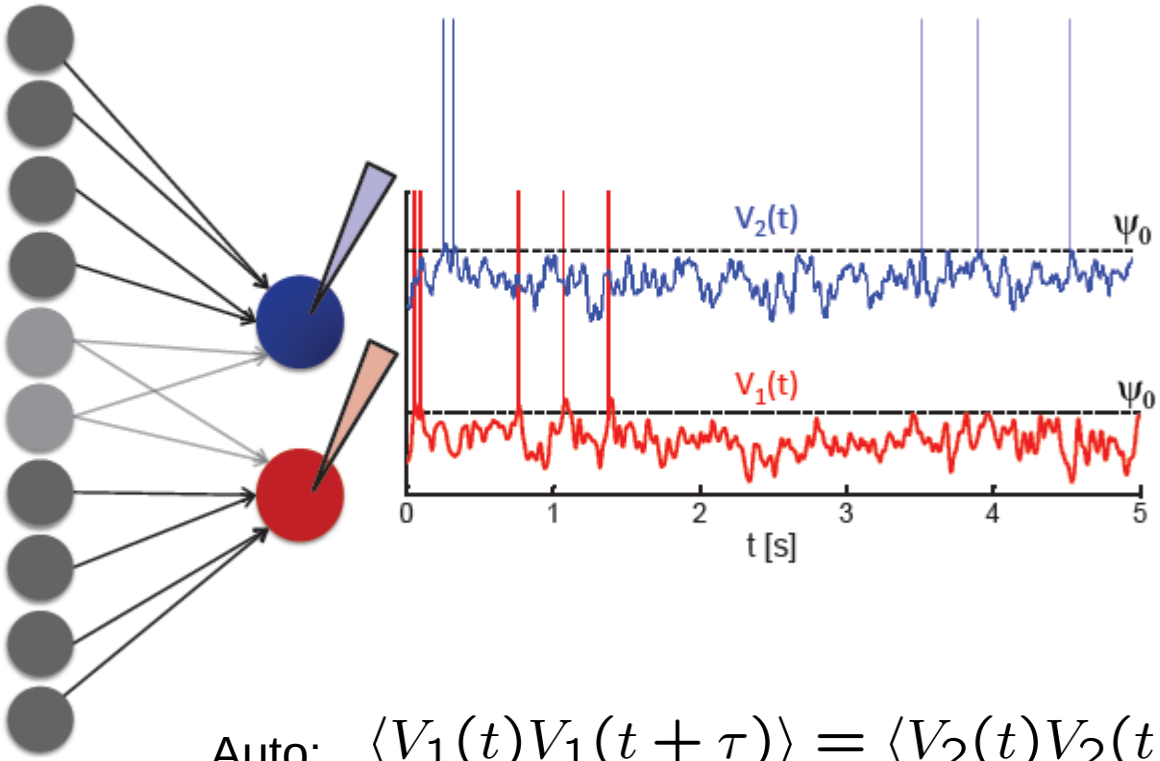
$$\nu_{cond}(\tau) = \frac{1}{\sqrt{\nu_1 \nu_2}} \langle s_1(t) s_2(t+\tau) \rangle$$



Firing rate of neuron a time lag τ after the spike of neuron

Tchumatchenko, Malyshev, Geisel, Volgushev, Wolf

Two neurons



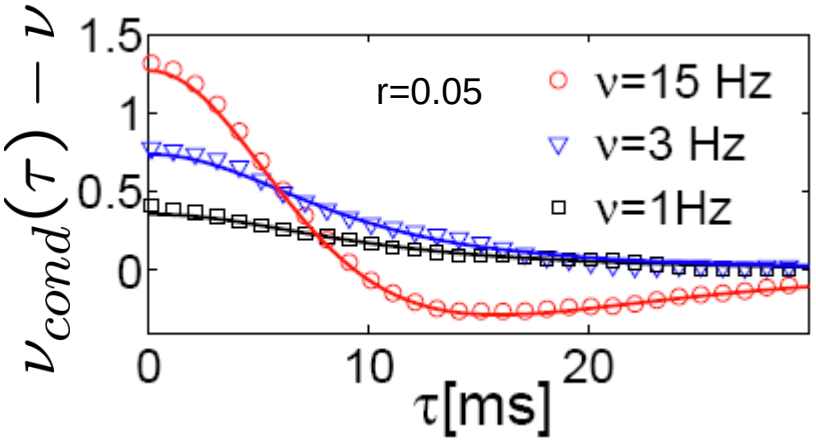
Auto: $\langle V_1(t)V_1(t+\tau)\rangle = \langle V_2(t)V_2(t+\tau)\rangle = C(\tau)$

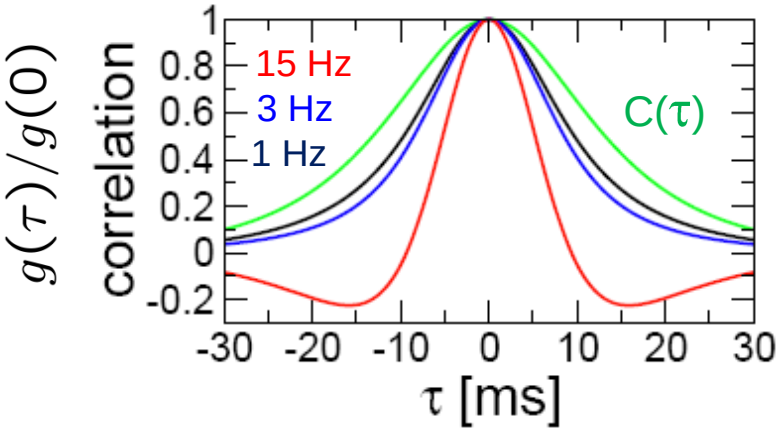
Cross: $\langle V_1(t)V_2(t+\tau)\rangle = rC(\tau)$

Weakly correlated neurons

$$\nu_{cond}(\tau) = \nu + r\nu \underbrace{\left(\frac{C(\tau)\psi_0^2}{\sigma_V^4} - \frac{\pi C''(\tau)}{2\sigma_{\dot{V}}^2} \right)}_{g(\tau)} + O(r^2)$$

$g(\tau)$ Correlation gain





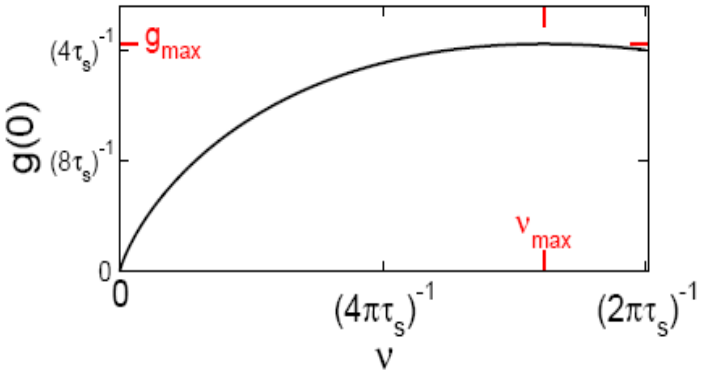
Spike correlations are firing rate dependent

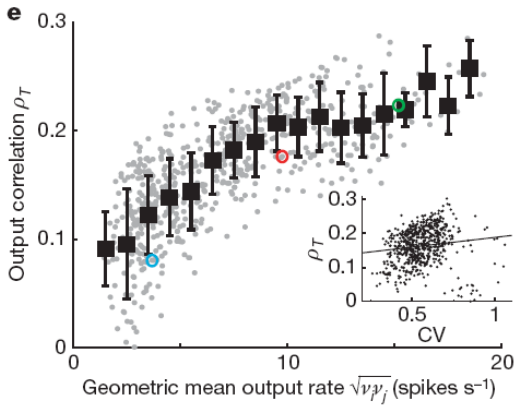
…which was an unexplained experimental result
I. Lampl et al., *Neuron*, 22:361–374, (1999)

Weakly correlated neurons

$$\nu_{cond}(\tau) = \nu + r\nu \underbrace{\left(\frac{C(\tau)\psi_0^2}{\sigma_V^4} - \frac{\pi C''(\tau)}{2\sigma_{\dot{V}}^2} \right)}_{g(\tau)} + O(r^2)$$

$$g(0) = \nu(-2\log(2\pi\nu\tau_s) + \pi/2)$$

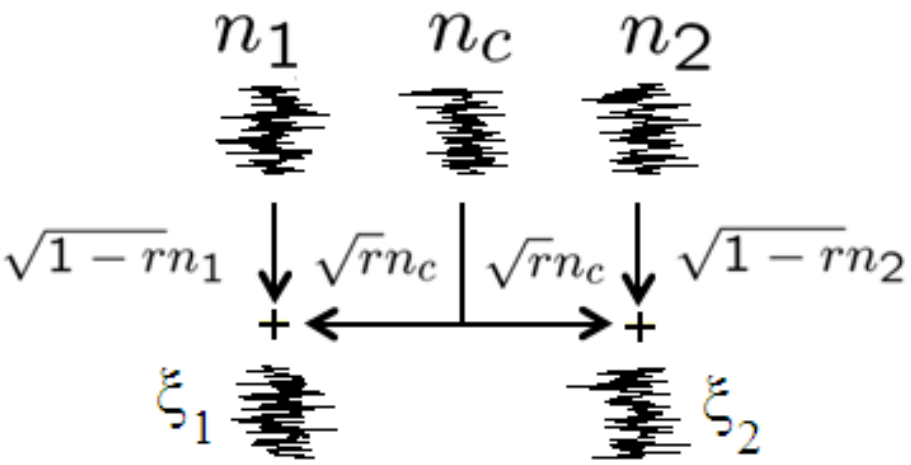


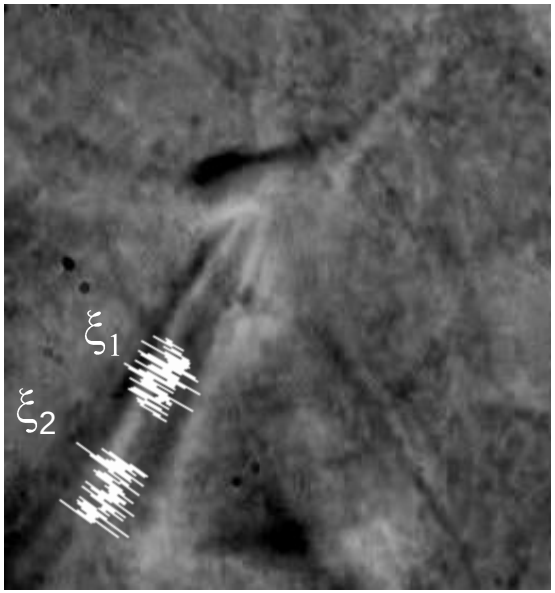


More spikes → more correlation

J. de la Rocha et al., *Nature*, 448:802–806, (2007)

In vitro L2/3 pyramidal neurons





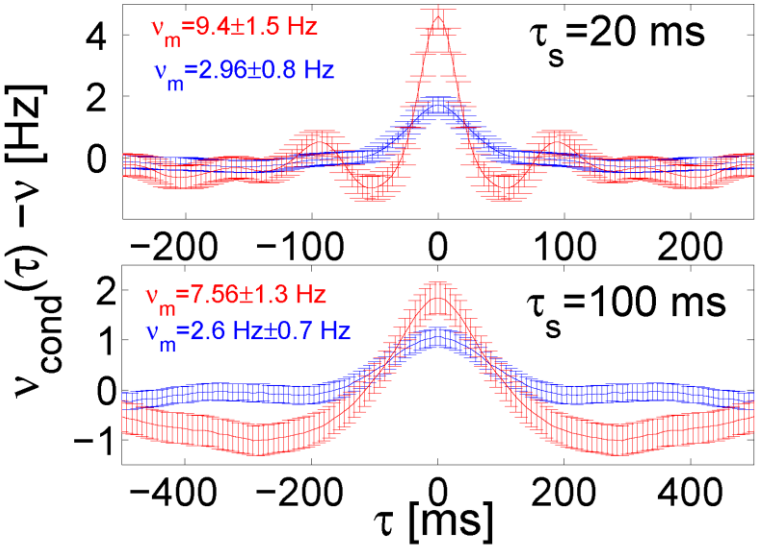
$$\xi_j(t) = \sqrt{1-r}n_j(t) + \sqrt{r}n_c(t)$$

Experimental test

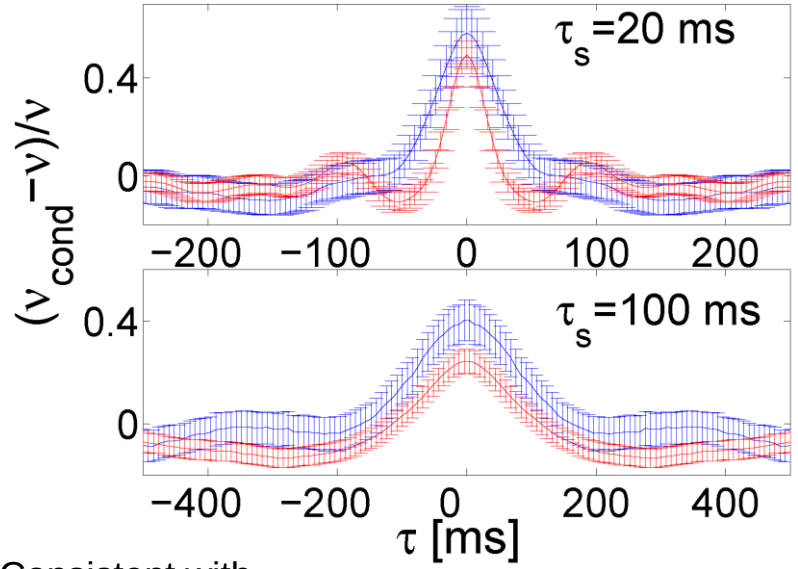
$$\nu_{cond}(\tau) = \nu + r\nu\left(\frac{C(\tau)\psi_0^2}{\sigma_V^4} - \frac{\pi C''(\tau)}{2\sigma_{\dot{V}}^2}\right) + O(r^2)$$

$$g(0) = \nu(-2\log(2\pi\nu\tau_s) + \pi/2)$$

Correlation gain increases with ν



Normalized correlation gain decreases with ν



Consistent with

J. de la Rocha et al., *Nature*, 448:802–806, (2007)

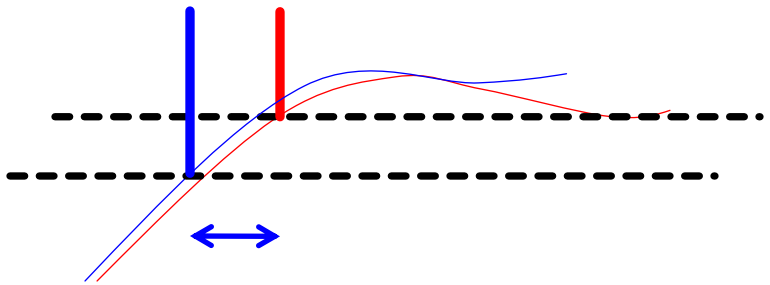
G. Svirskis and J. Hounsgaard, *Network*, 14:747–763, 2003

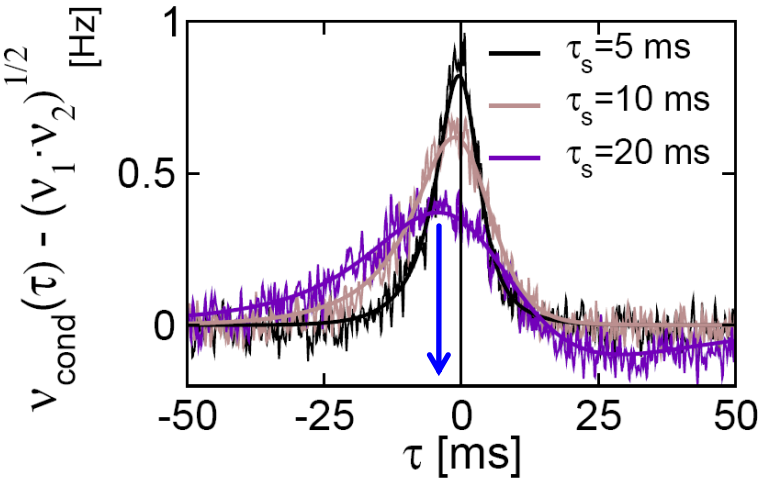
Firing rate differences

$$\nu_{cond}(\tau) = \sqrt{\nu_1\nu_2} + r \cdot g_{12}(\tau) - \boxed{C'(\tau)\frac{\tau_s \Delta}{\sigma_{V_1}\sigma_{V_2}\sqrt{2/\pi}}}$$

$$e_i = \frac{\psi_0}{\sigma_{V_i}},\ \Delta = (e_2 - e_1)$$

$$\tau_{Peak} = \frac{\sqrt{\pi/2}\tau_s\Delta/(e_1e_2)}{1+\pi(2e_1e_2\sigma_{V_1}\sigma_{V_2})^{-1}C^{(4)}(0)\tau_s^4}$$





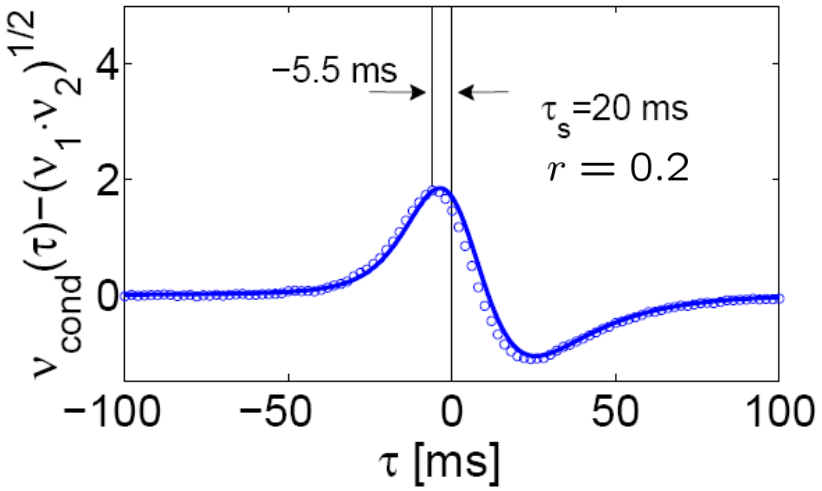
Difference in firing rates leads to asymmetry of spike correlations

$\boxed{\nu_1 = 0.1\ \text{Hz}}$ $\boxed{\nu_2 = 7\ \text{Hz}}$ r=0.05

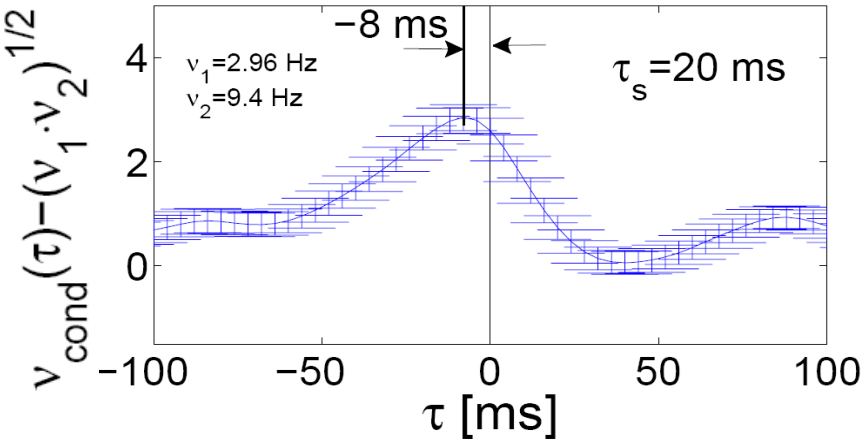
Tchumatchenko, Malyshev, Geisel, Volgushev, Wolf

Firing rate differences

Theory



In vitro



Summary I

- What determines pairwise spike correlations?

  Weak correlations: firing rate, correlation time and correlation strength

- How is temporal structure of input currents reflected in the spike correlations?

  Mixture of $C(\tau)$ and $C''(\tau)$

  Temporal width of spike correlations is firing rate dependent

- What is the correlation structure in a pair with different firing rates?

  Spike correlations are asymmetric

Does this hold for strong correlations?

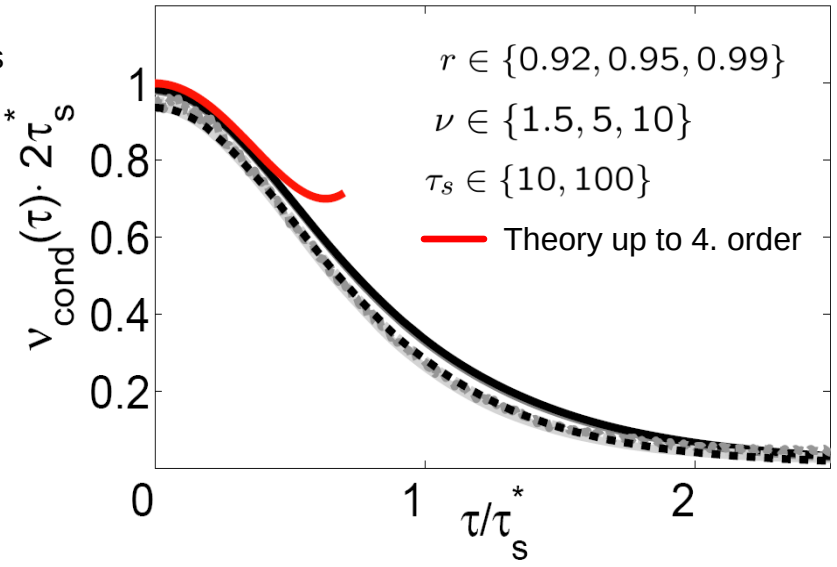
Strongly correlated neurons

$$\nu_{cond}(\tau) = \frac{1}{2\tau_s^*} - \frac{3}{4\tau_s^*}\left(\frac{\tau}{\tau_s^*}\right)^2 + \frac{30}{32\tau_s^*}\left(\frac{\tau}{\tau_s^*}\right)^4 + O(\tau^6)$$

$$\tau_s^* = \sqrt{2}\sqrt{1-r}\tau_s$$

1. Threshold independence
2. Temporal fidelity is decreasing with τ_s
3. Universal rescaling

Simulations and Analytics



consistent with previous findings

Z. F. Mainen and T. J. Sejnowski, Science, 268:1503–1506, 1995

Strongly correlated neurons

$$\nu_{cond}(\tau) = \frac{1}{2\tau_s^*} - \frac{3}{4\tau_s^*}\left(\frac{\tau}{\tau_s^*}\right)^2 + \frac{30}{32\tau_s^*}\left(\frac{\tau}{\tau_s^*}\right)^4 + O(\tau^6)$$

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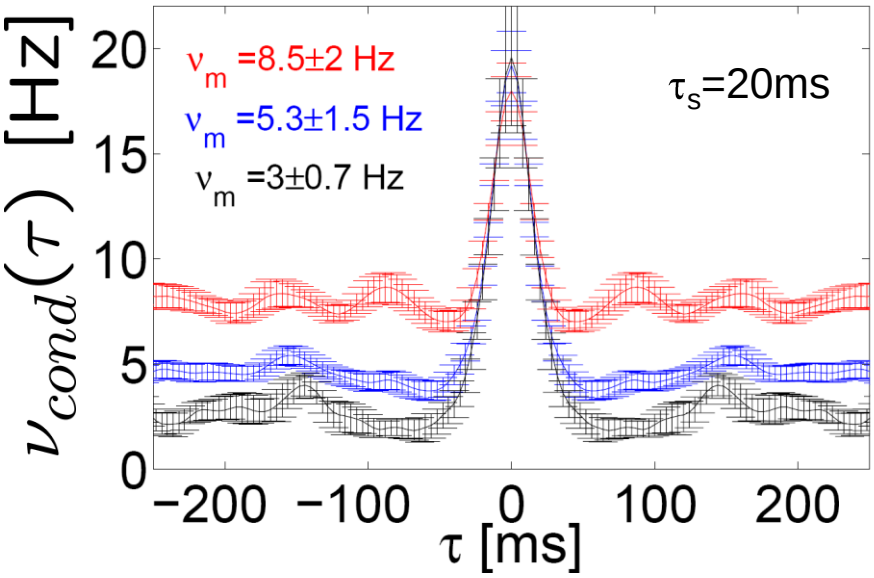
In vitro



Strong spike correlations are firing rate independent

Summary&Discussion

- Weak spike correlations depend on firing rate and voltage correlation function

  The synaptic time constants (AMPA, NMDA etc.) determine spike correlations
  Changes in firing rates (sensory stimulus) can effect the decoding schemes

- Temporal width of weak spike correlations is firing rate dependent

  Synchrony on shorter time scales (e.g. AMPA vs. NMDA) is more likely for higher firing rates

- Strong spike correlations depend on correlation time but not on firing rate

  Spike timing reliability in a network depends on the synaptic time constants

- Asymmetric spike correlations in a pair with different firing rates

  Delays in firing do not imply directed connectivity nor delayed inputs
  STDP could occur in firing rate asymmetric pairs

Thank you for your attention!