Hydrodynamic Objects:

Localization, Feature Extraction, and Recognition

Andreas B. Sichert and J. Leo van Hemmen

Physik Department & Bernstein Center for Computational Neuroscience, TU München

abs@tum.de, lvh@tum.de

Abstract

All fish and some aquatic amphibians possess a unique sensory facility, the *lateral line*. The lateral-line system is a mechanosensory facility of aquatic animals that enables them not only to localize prey, predator, obstacles, and conspecifics, but also to *recognize* hydrodynamic objects. We present on the one hand a mathematical description of the characteristics fish print into the flow field and on the other hand we introduce an explicit model explaining how the lateral line can use the imprinted information to distinguish differently shaped submerged moving objects. Our model uses the unambiguous set of multipole components to identify the corresponding hydrodynamic object. Furthermore, we show that within the natural range of *one fish length* the velocity field contains far more information than that due to a *dipole*. Finally, the model we present is easy to implement both neuronally and technically, and agrees well with available neuronal, physiological, and behavioral data on the lateral-line system.

Mathematical Description

The velocity field $\mathbf{v} = (v_r, v_\varphi, v_\theta) = -\nabla\Phi$ represents an adequate and natural stimulus to the lateral-line system. It can be described by a multipole expansion of the velocity potential Φ. Using real spherical harmonics Y^R_{lm} and multipoles $\mathbf{q} = (..., q_{lm}, ...)$ we can expand the velocity potential Φ,

$$\Phi = \sum_{l,m} q_{lm}\Phi_{lm} = \sum_{l,m} q_{lm}\frac{1}{2l+1}\frac{Y^R_{lm}(\theta,\varphi)}{r^{l+1}} \, . \tag{1}$$

The above equations can be written in matrix form $\mathbf{v} = \mathcal{A}\mathbf{q}$. In the Euler description of hydrodynamics we can specify $w := \mathbf{v}_\infty \cdot \mathbf{n}$ as the normal velocity component at the boundary of a submerged object moving with $\mathbf{v}_\infty$.

Sampling the physical problem $\mathcal{T} := (\ldots, \mathbf{n}_i{}^T\mathcal{A}_i, \ldots)$ we derive a matrix equation $\mathcal{T}\mathbf{q} = \mathbf{w}$. We can now approximate the multipole moments in a least-square sense

$$\mathbf{q} = (\mathcal{T}^T\mathcal{T})^{-1}\mathcal{T}^T\mathbf{w} \, . \tag{2}$$

How, then, can aquatic animals such as fish reconstruct the above multipole moments from water velocity measured through their lateral-line system?

We define the likelihood function L(q,$\mathbf{r}_0$) of finding the correct multipole moments $\mathbf{q}$ at the correct position $\mathbf{r}_0$ of the submerged moving object (SMO) with respect to the lateral-line organs of the detecting animal (DA) at $\mathbf{r}_i$. Maximizing L(q,$\mathbf{r}_0$) with respect to $\mathbf{q}$ leads to

$$\frac{\partial L}{\partial \mathbf{q}}(\mathbf{q}, \hat{\mathbf{r}}_0)\,|_{\mathbf{q}=\hat{\mathbf{q}}} = \left[(\mathcal{T}^T\mathcal{T}+\sigma^2)\,\mathbf{q} - \mathcal{T}^T\mathbf{w}\right]|_{\mathbf{q}=\hat{\mathbf{q}}} = 0 \, , \tag{3}$$

a positional map. L(q,$\mathbf{r}_0$) is maximal at the right position $\hat{\mathbf{r}}_0$ of the SMO and computes the correct $\hat{\mathbf{q}}$. At $\hat{\mathbf{r}}_0$ the multipoles can be computed by means of a matrix equation, $\hat{\mathbf{q}} = (\mathcal{T}^T\mathcal{T}+\sigma^2)^{-1}\mathcal{T}^T\mathbf{w}$. Here $\mathcal{T}$ depends on $\mathbf{r}_0$ and $\mathbf{r}_i$.

Differently Shaped Objects

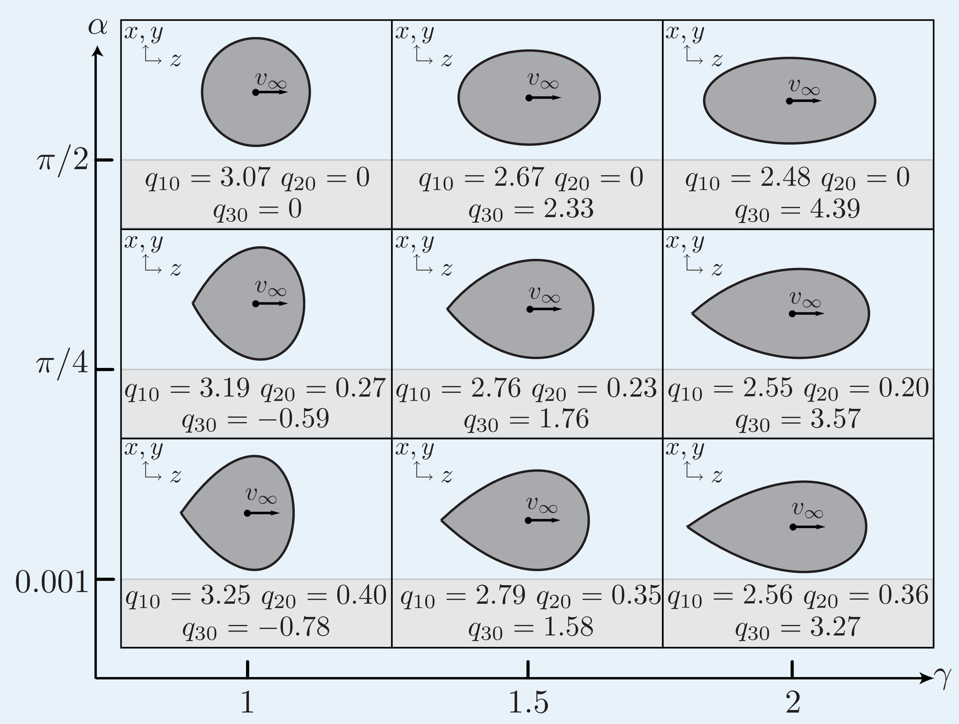
An important category of hydrodynamic objects are submerged moving objects (SMO). As illustrated in the cartoon, it can be crucial to survival being able to distinguish them by means of their flow-field characteristics.

For the sake of simplicity we focus on rotationally symmetrical bodies whose surface S can be described through coordinates $\varsigma \in [0,\alpha]$ and $\eta \in [0, 2\pi]$.

$$S(\varsigma,\eta) = \cos\left(\frac{\varsigma\pi}{\alpha 3}\right)\begin{pmatrix} \frac{\gamma^{-1/3}}{\sqrt{2}\sin(\alpha/2)}\sin(\varsigma)\cos(\eta) \\ \frac{\gamma^{-1/3}}{\sqrt{2}\sin(\alpha/2)}\sin(\varsigma)\sin(\eta) \\ \gamma^{2/3}\cos(\varsigma) \end{pmatrix} \tag{4}$$

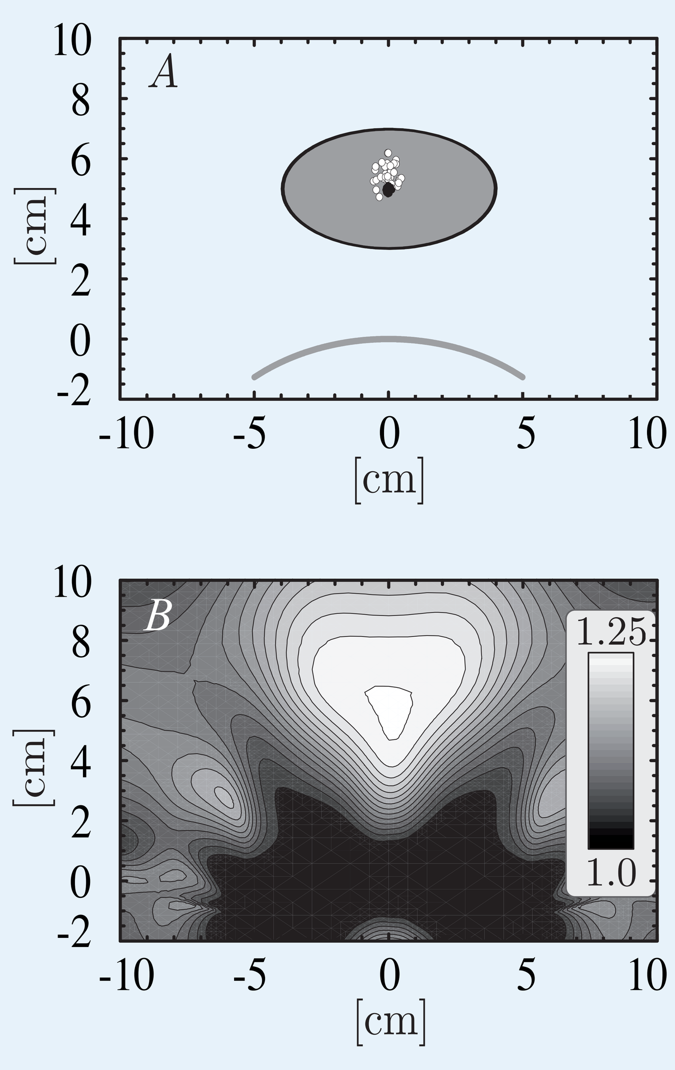
The two parameters $\alpha \in (0, \pi/2]$ and $\gamma \in (0,\infty)$ determine the *shape* of an SMO. Due to Eq. (2) we can calculate the multimole moments $\mathbf{q}$ corresponding to the shapes. Different shape parameters result in distinguishable sets of multipole components.

We conclude, that if we are able to reconstruct the multipoles from the flow field measured at the lateral line we can distinguish different shape parameters and, thus, shapes. So our little fish in the cartoon could recognize what is behind it even without sight.



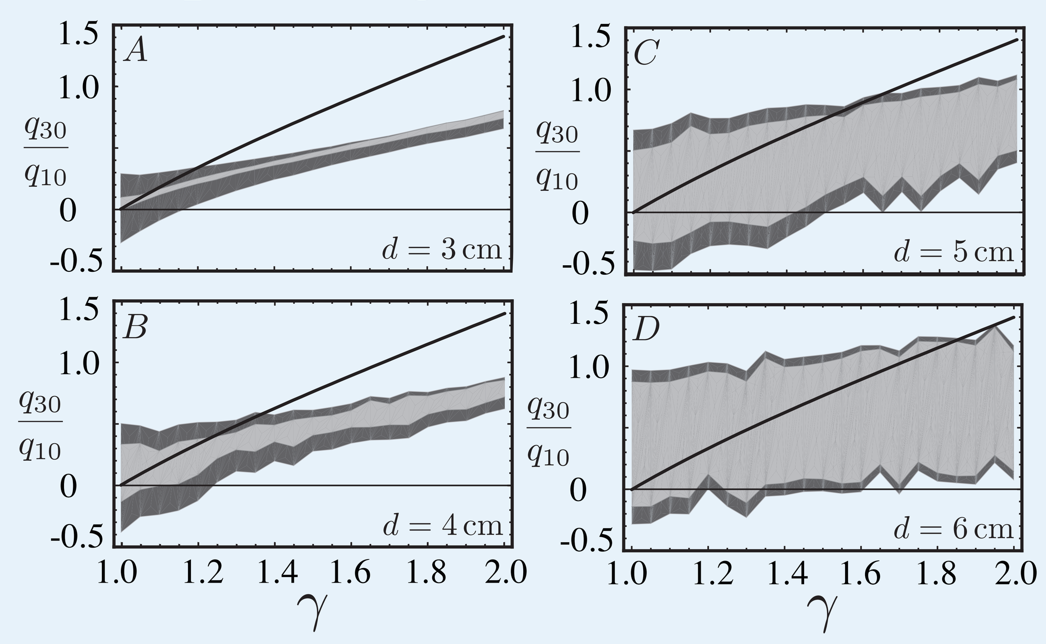
Multipoles (dipole, quadrupole, octupole) calculated for different shape parameter (4) corresponding to different SMOs moving with v_∞.

Localization Performance



Using the dipole term only, we can easily reconstruct the SMO's position (*A*, cloud of black points). If we take *only* three multipole moments as an *efficient* minimal model, our method allows a proper estimate of *both* position (*A*, small open circles) and form; see right graph. The graph *B* depicts the likelihood L(q,$\mathbf{r}_0$) corrsponding to the neuronal map of position estimation (3).

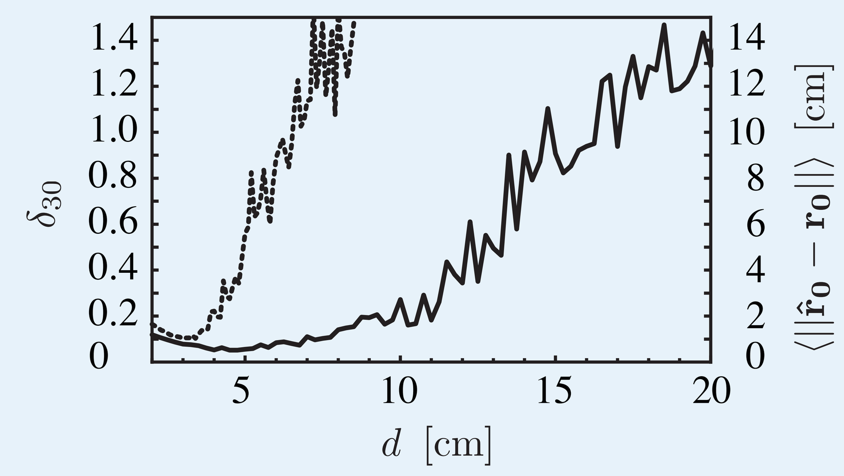
Shape Recognition Performance



Reconstructed normalized octupole strength q_{30}/q_{10} for $\alpha=\pi/2$ and different shapes parameters γ (4) at four distances *d*. Since we have used only three multipoles, q_{30} differs from the real q_{30} (black line). The gray-colored region depicts the value of q_{30} with noise at the most likely position (light grey) and at wrong estimated positions (dark grey). Object recognition works properly up to distances compareable to the lenght of the SMO.

Two Characteristic Length Scales

The relative reconstruction error δ_{30} (dashed line) of the multipole component q_{30} shows that for distances *d* larger than a *moving object's size* (5 cm) reconstructig $\mathbf{q}$ is not possible. The solid line depicts the localization error $\langle||\hat{\mathbf{r}}_0 - \mathbf{r}_0||\rangle$ which starts to grow fast at a distance comparable to the *length of the detecting animal* (10 cm).



So we can identify clearly two different length scales that are important for lateral line perception.

Conclusion and Further Reading

Within the natural range of *one fish length* the velocity field contains far more information than that due to a dipole (i.e., sphere). Our method can reconstruct shape information at distances compareable to the body length of an SMO. The model agrees well with biological findings and extends the present theoretical understanding of hydrodynamic object perception through the lateral line. Finally, these findings can also be applied to biomimetics, e.g., to improve passive naval navigation systems.

Further information can be found in the following references:

[1] A.B. Sichert, R. Bamler, and J.L. van Hemmen, Phys. Rev. Lett. (2009) **102**:058104
[2] A.B Sichert and J.L. van Hemmen, Biol. Cybern (2009) submitted
[3] J.-M.P. Franosch, A.B. Sichert M.D. Suttner, and J.L. van Hemmen, Biol. Cybern (2005) **93**:231
[4] H. Lamb, Hydrodynamics, 6th ed. (Cambridge University Press, 1932)

ABS was funded by BCCN - Munich.