

Relating evoked and spontaneous activities in a generative modeling framework

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Statistical structure of neural activities

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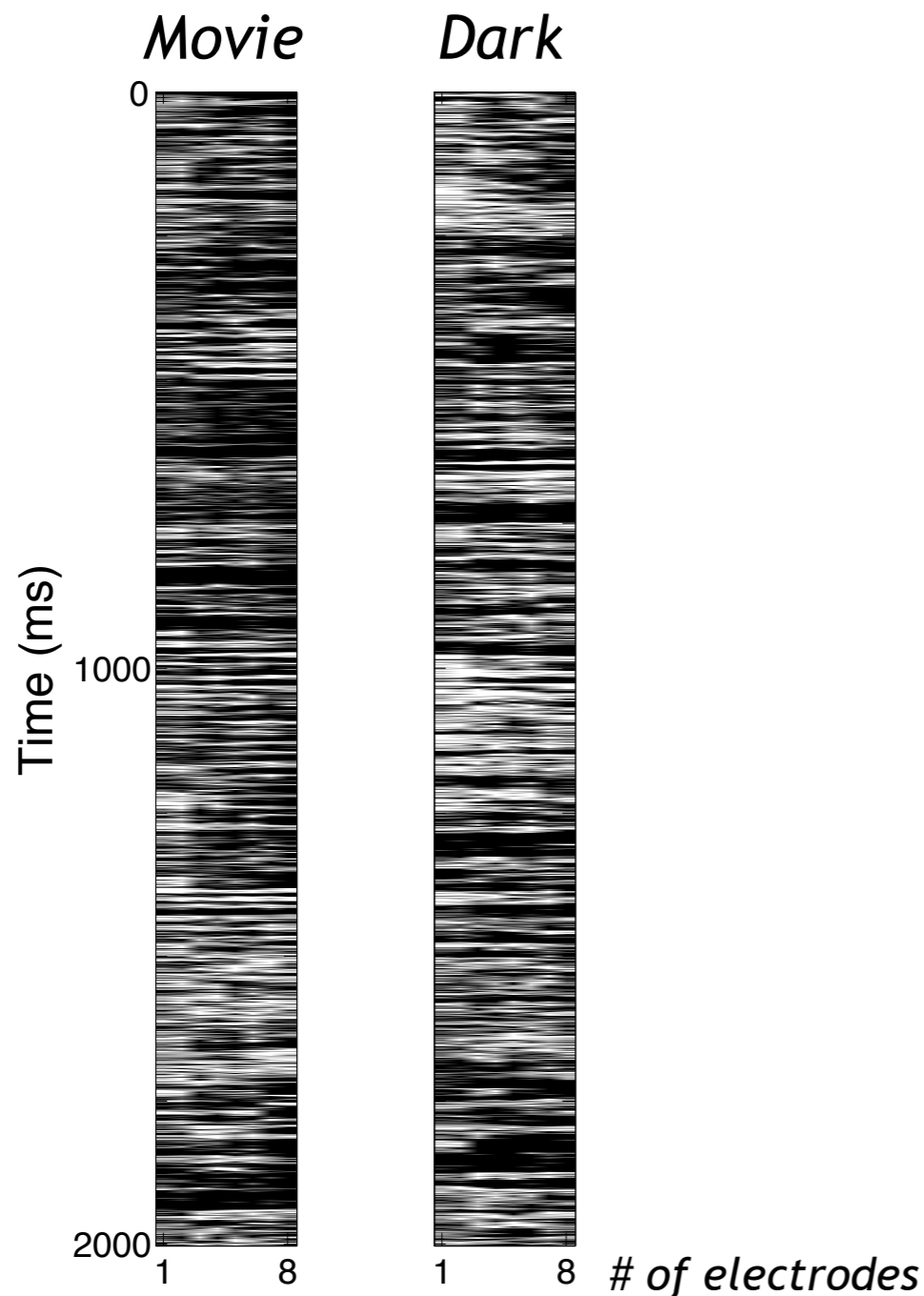
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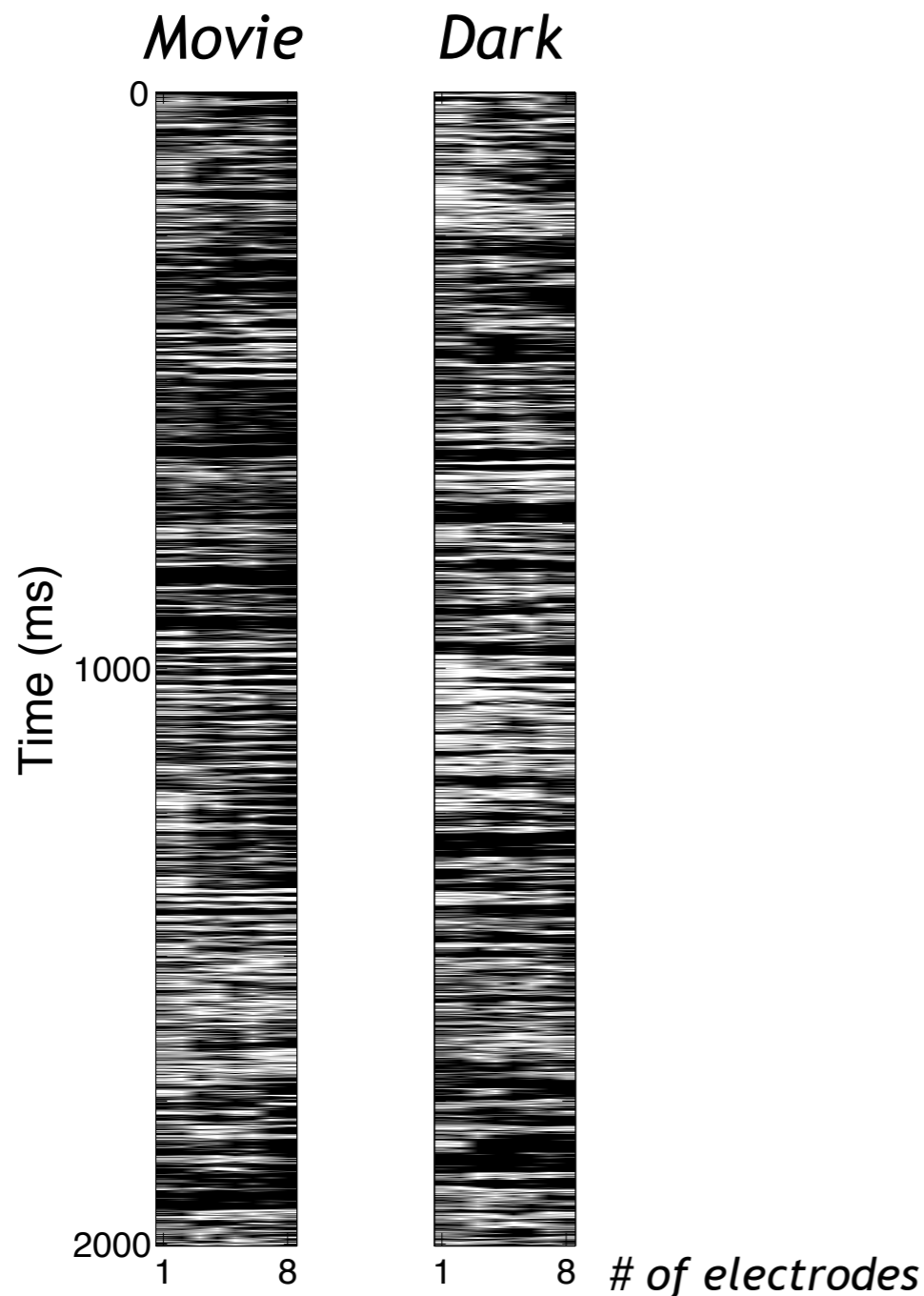
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- we need to assess the differences between high-dimensional multivariate distributions

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- maximum entropy models
 - handles the whole distribution of activities
 - level of correlations needs to be determined a priori
 - number of parameters grows exponentially

How to compare distributions of activities?

Kullback-Leibler divergence

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- *LNP models*: $P(y)$ is not available, only $P(y | x)$, therefore the stimulus ensemble has to be marginalized
- *max entropy models*: partition function for $P(y)$ needs to be determined

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- KL divergences are calculated with respect to a reference condition

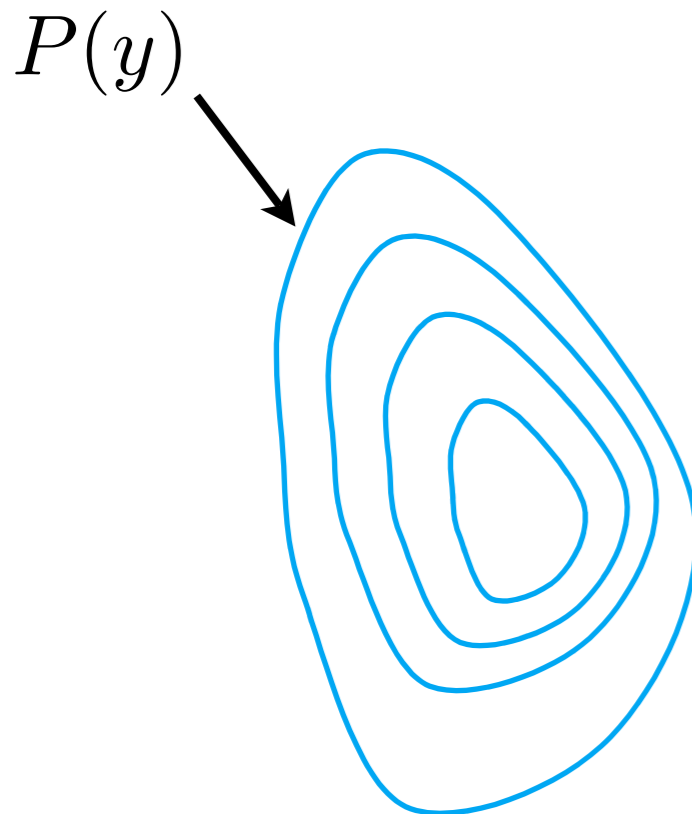
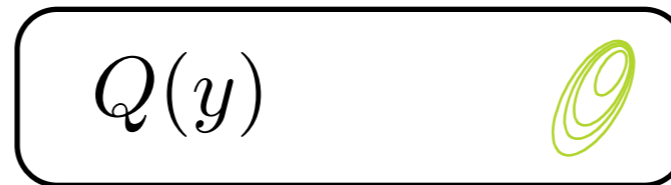
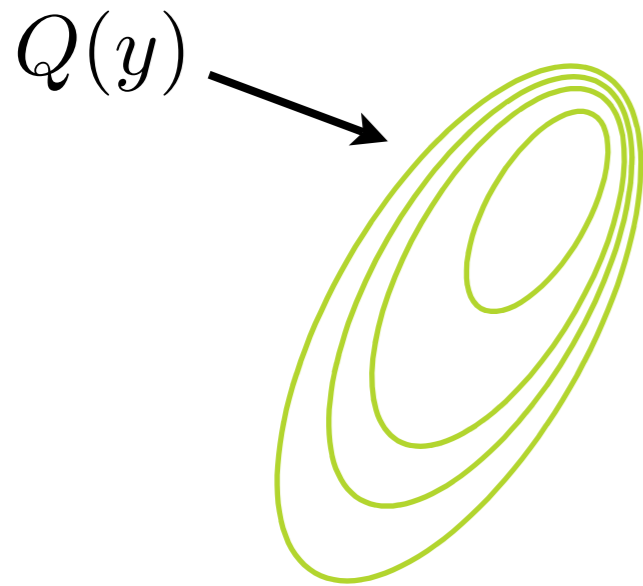
$$\begin{aligned} \text{KL}[P_{\text{ref}}(y), P_1(y)] - \text{KL}[P_{\text{ref}}(y), P_2(y)] = \\ \text{CE}[P_{\text{ref}}(y), P_1(y)] - \text{CE}[P_{\text{ref}}(y), P_2(y)] \end{aligned}$$

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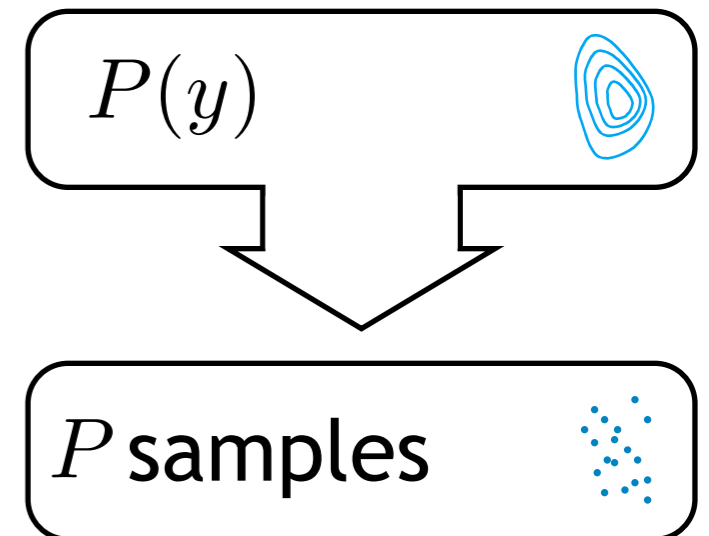
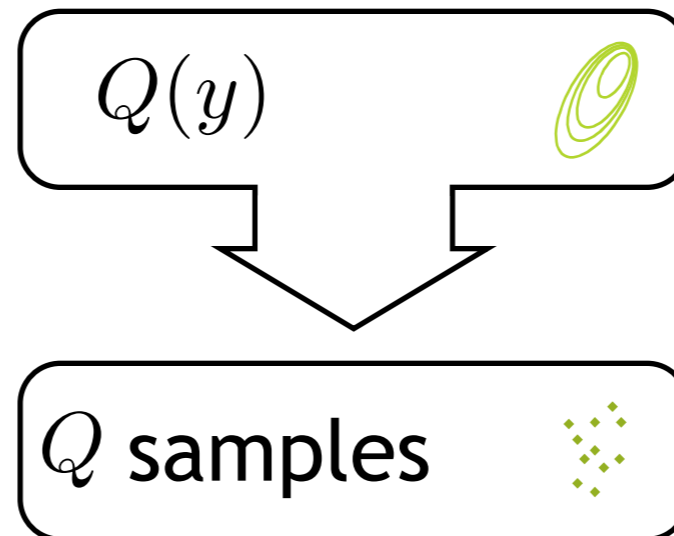
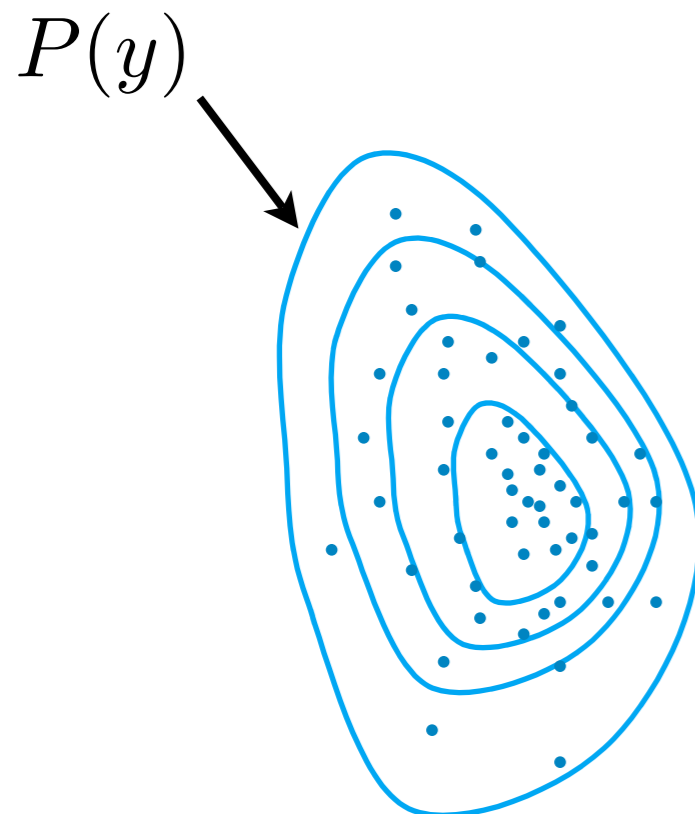
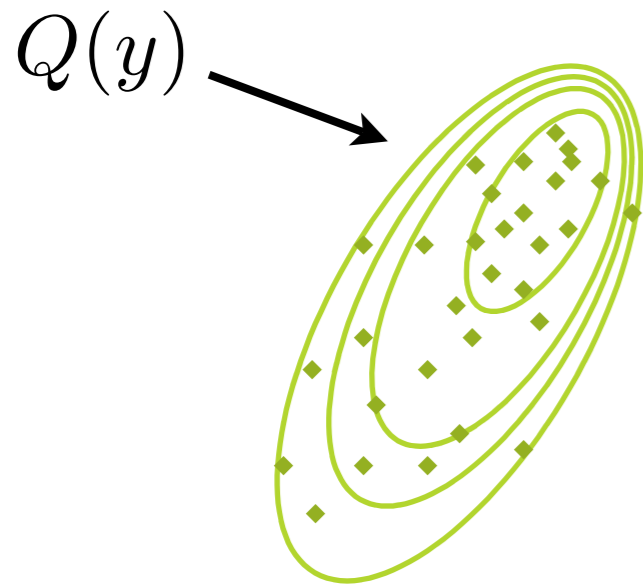
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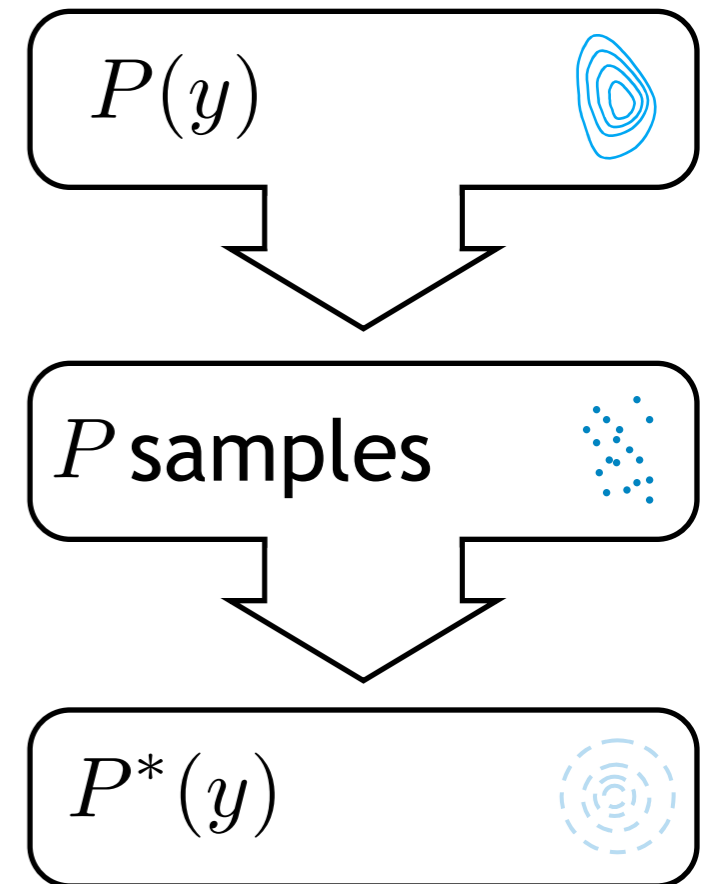
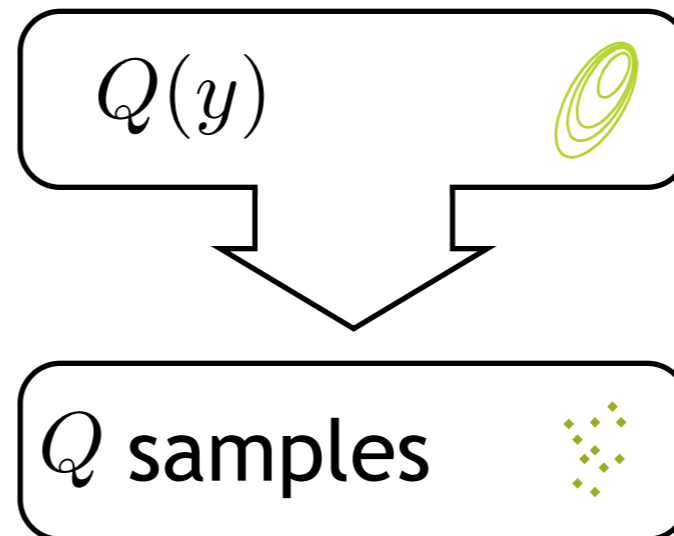
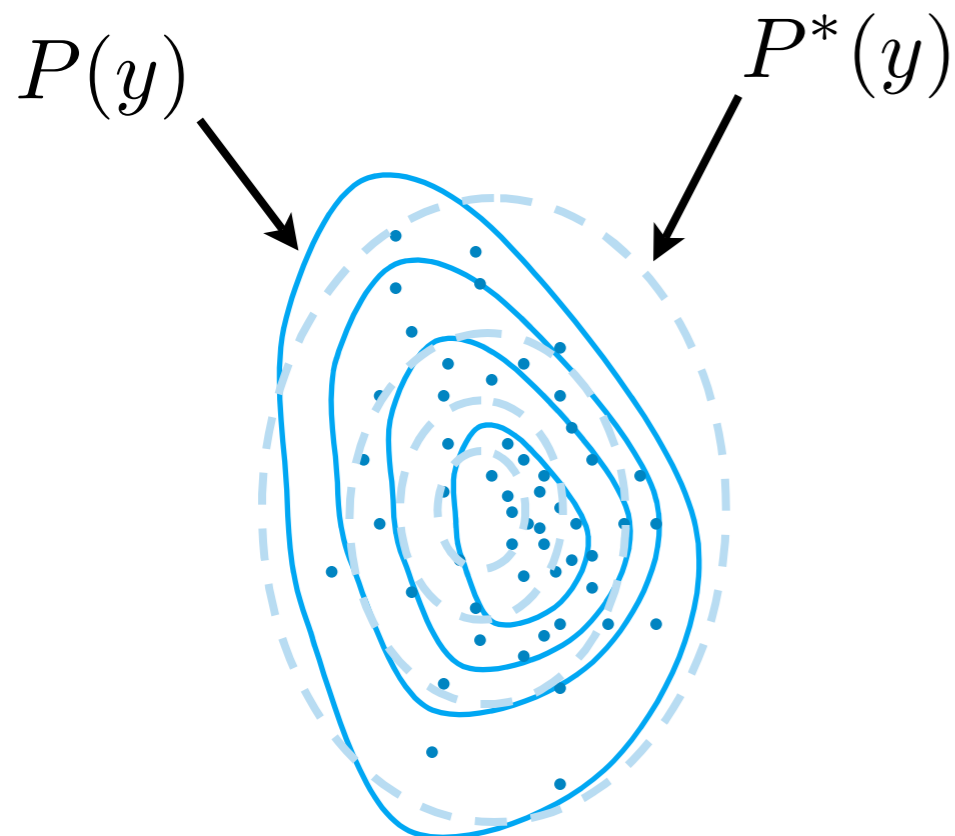
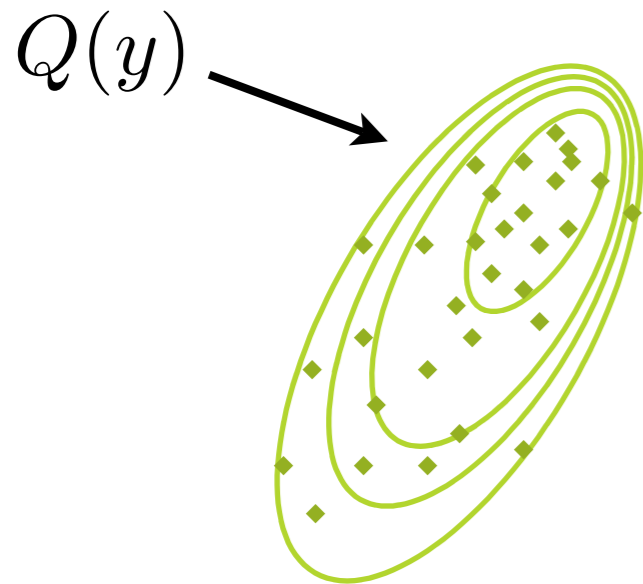
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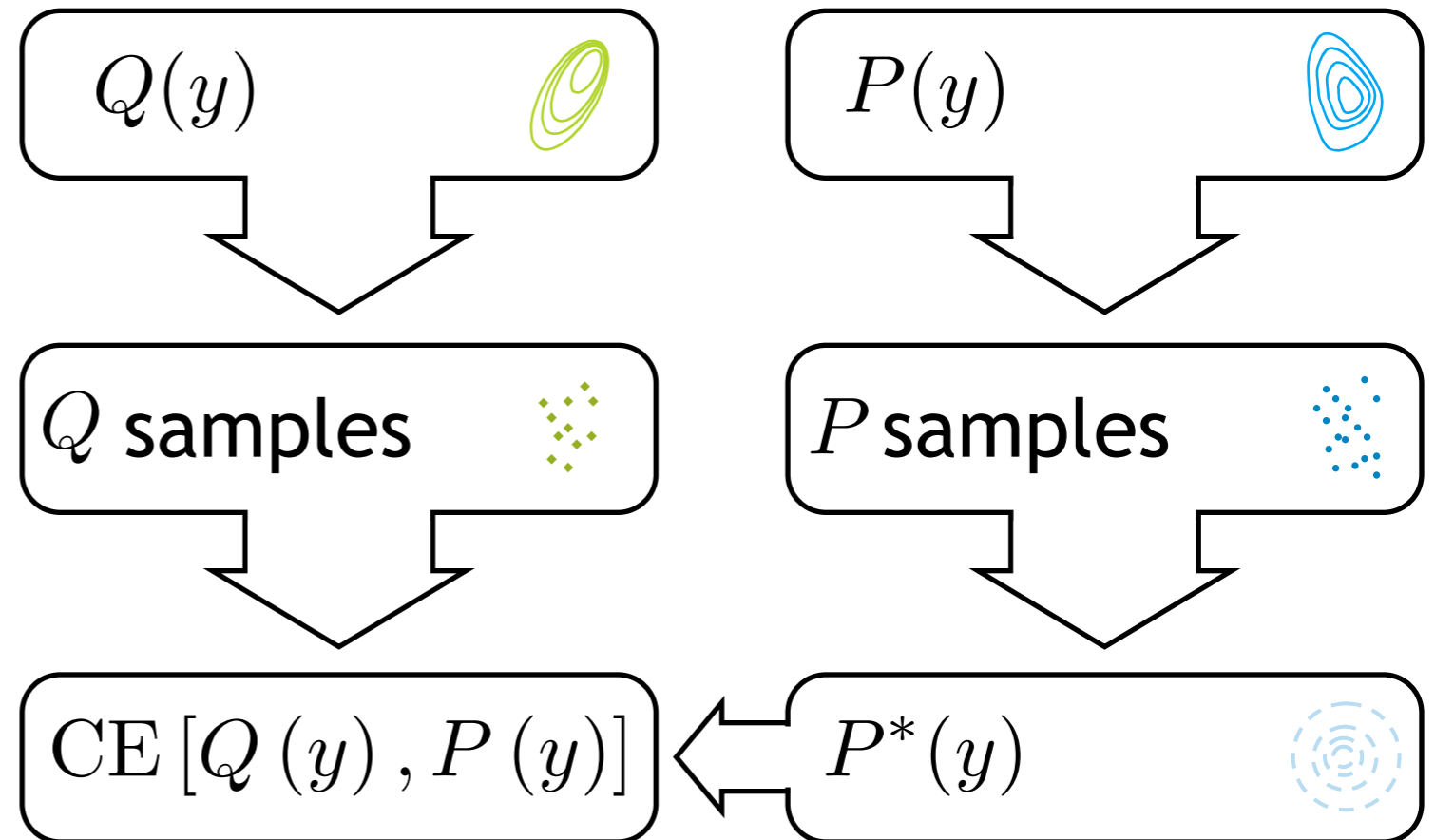
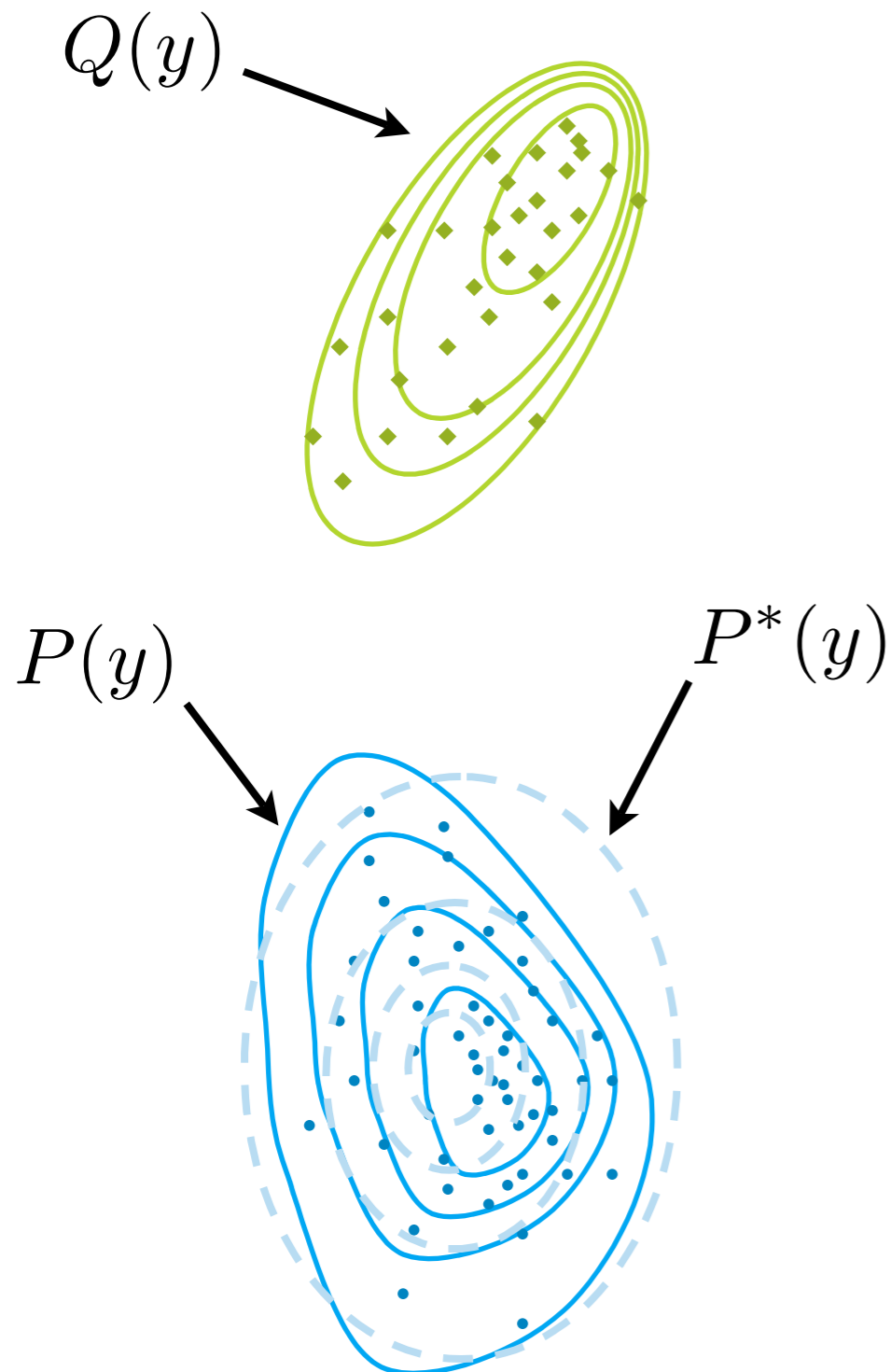
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Demonstration on synthetic data

Evaluating SA and EA distributions

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- **Prediction:** Activity evoked by *natural images* is required to be more similar to spontaneous activity than that evoked by *noise*
- CE between natural image EA and SA
noise pattern EA and SA

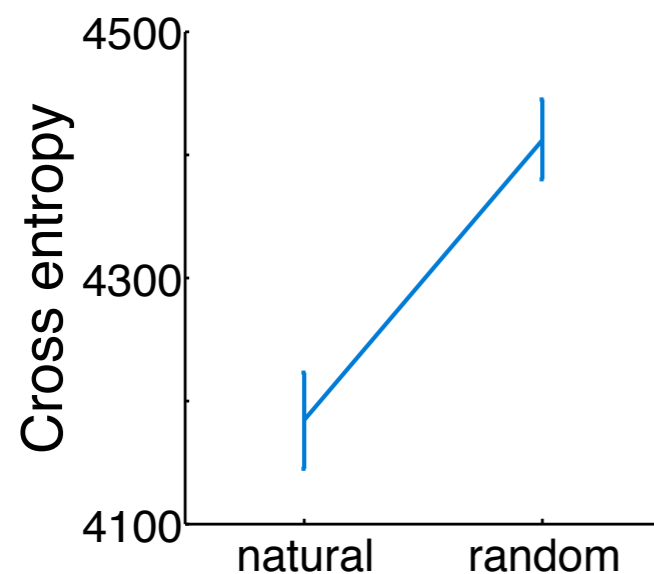
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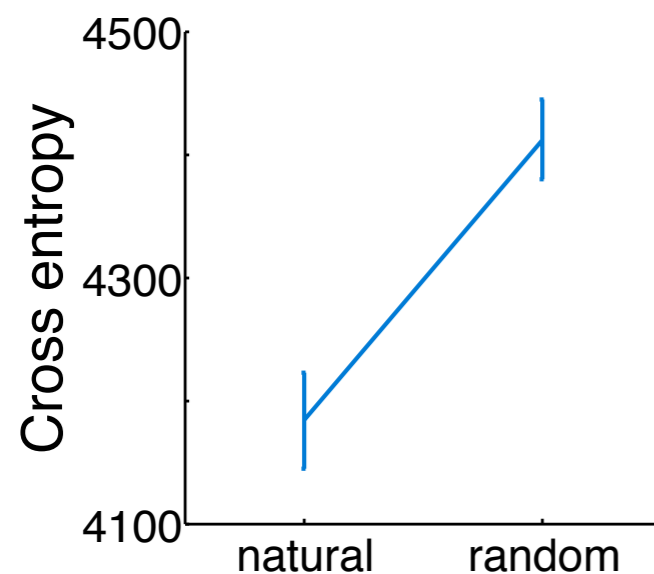
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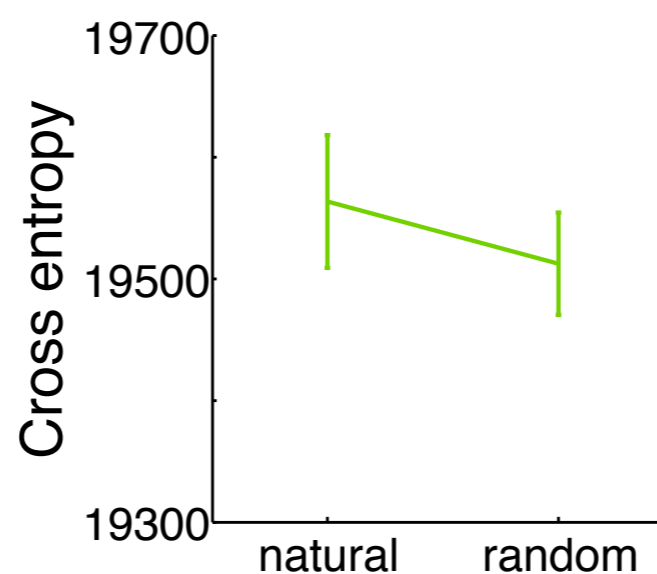
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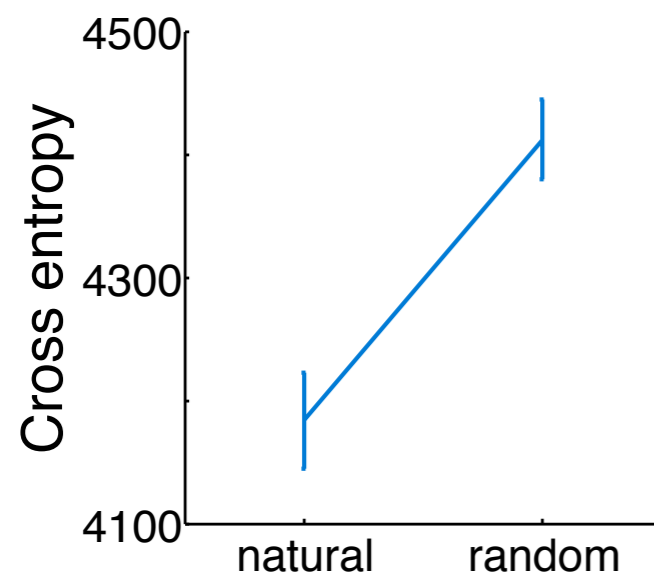
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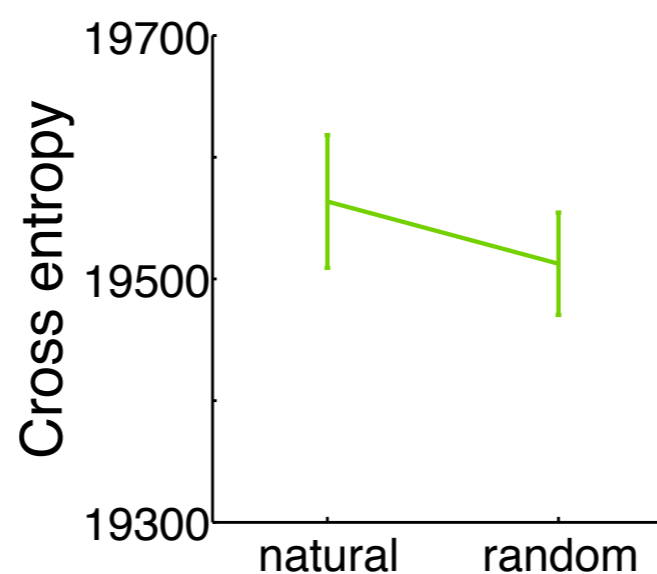
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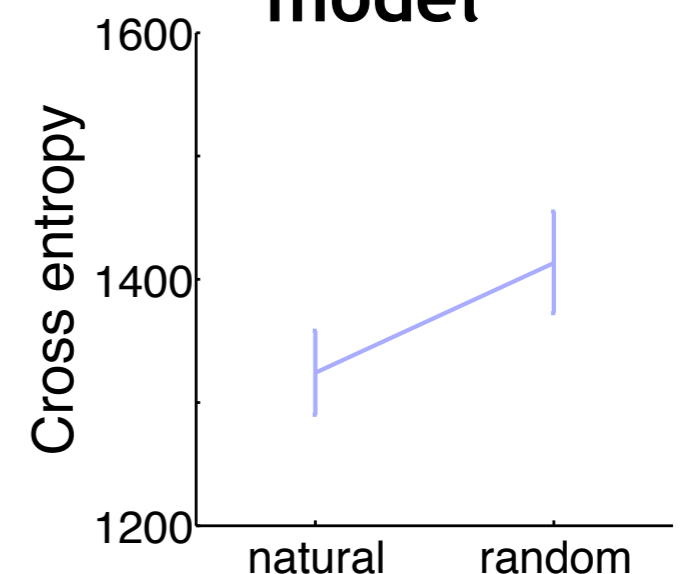
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Naive model



Partially observed model



Recorded data

EA and SA in the V1 of ferrets

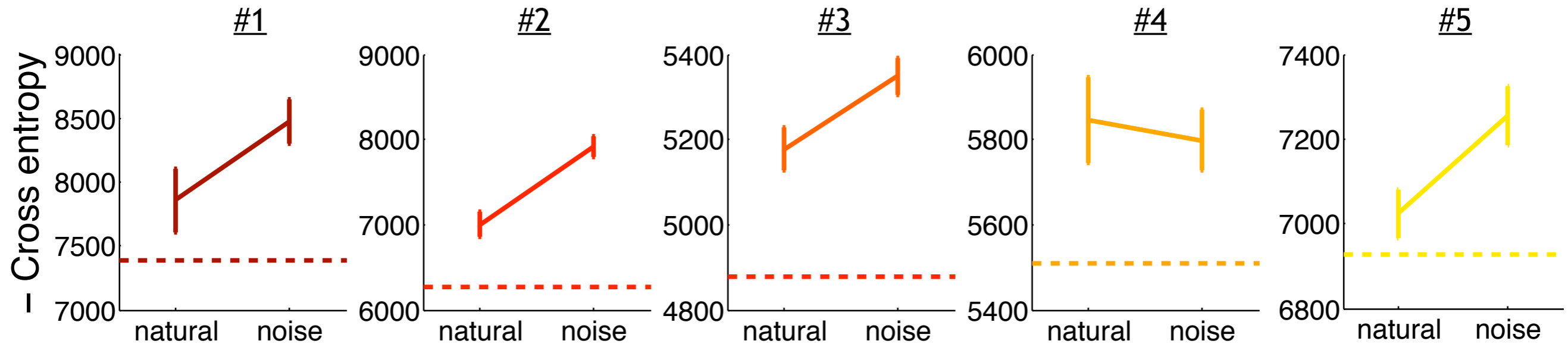
- recordings from adult behaving ferrets
- calculating CE between:
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Animals (n=5)



Conclusions

- KL with latent variable density estimators provide a general framework for quantifying the similarity between neural activities
- the method works even on limited number of neurons
- analysis of data from behaving animal is consistent with model predictions
- further analysis is needed to assess whether dark condition or sleep is a better model for spontaneous activity

Acknowledgements

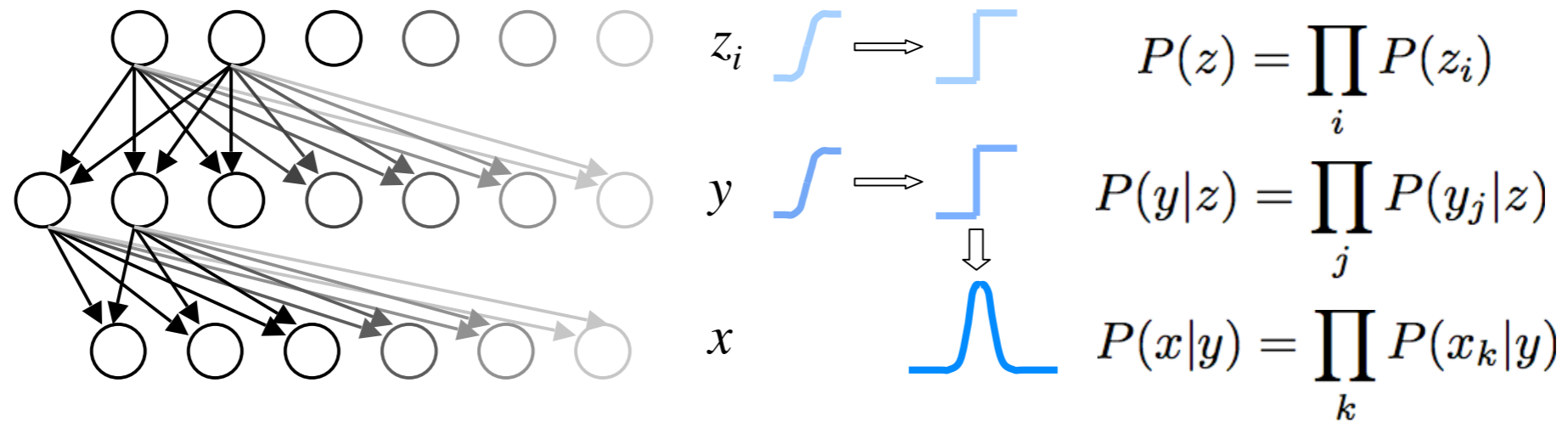
József Fiser (Brandeis University)

Pietro Berkes (Gatsby Unit, UCL)

Máté Lengyel (University of Cambridge)

Swartz Foundation

Validation on a model trained with natural images



Evoked activity (EA)	$P_{\text{EA}}(y x)$	$\int P(y x, z) P(z) dz \propto \int P(x y) P(y z) P(z) dz$
Marginalized EA	$P_{\text{mEA}}(y)$	$\int P_{\text{EA}}(y x) P(x) dx$
Spontaneous activity (SA)	$P_{\text{SA}}(y)$	$\int P(y z) P(z) dz$