Judgment

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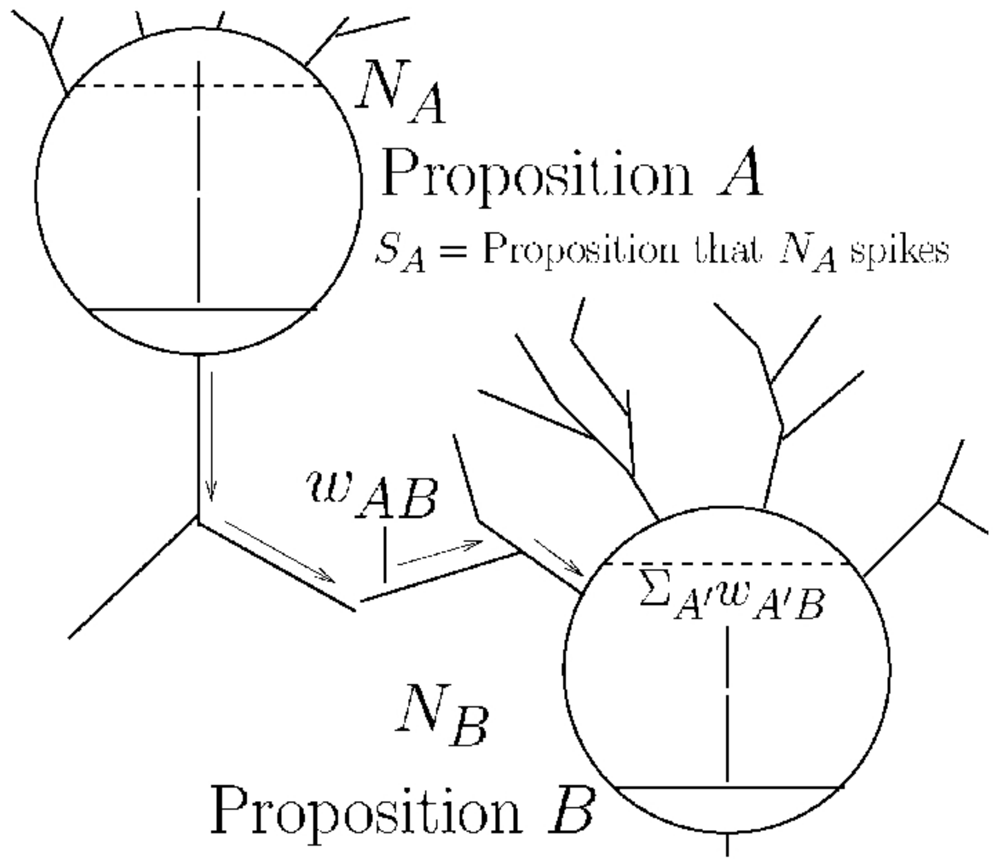
Approx. Linear Summation of Inputs



Conjecture: w_{AB} corresponds to an amount of evidence or information relating proposition A to proposition B

Conjecture: Spike Indicates a Logical Judgment

When a judgment occurs, a proposition is *given*.

Circumstance 1	D is judged	Circumstance 2
A, B, C are given	$\longrightarrow$	A, B, C, D are given
Every proposition, H, can be assigned a probability, $P(H)$	to be true	$P_{\text{New}}(H) = P_{\text{Old}}(H \vert D)$

A *judgment* is distinct from an *assertion* (statement about a single circumstance). E.g.

(i) All ravens are black
(ii) All non-black things are non-ravens

(i) and (ii) are equivalent as statements, but different as judgments.

$$\text{All } \underbrace{\text{ravens}}_{\text{subject}} \text{ are } \underbrace{\text{black}}_{\text{predicate}}$$

$P(x$ is a raven) is unchanged by this judgment
$P_{\text{New}}(x$ is black) $= P_{\text{Old}}(x$ is black OR x is a raven)

Information and Propositions

A - Proposition $\qquad$ $P(A)$ - Probability of A

$i(A) = \log \frac{1}{P(A)} =$ Amt. of info. provided if A is given

$=$ Min. amt. of info. which must be given before $P(A)$ can reach 1

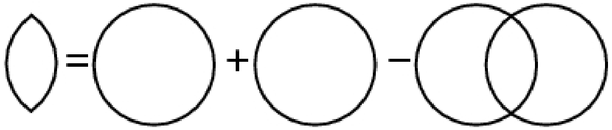
$$P(A|B) = 1 \Rightarrow P(B) = P(AB) \leq P(A) \Rightarrow i(B) \geq i(A)$$

$i(A) =$ "Amount of information required to believe A"

$i(A \textit{ and } B \textit{ and } C) = i(A) + i(B) + i(C)$ if A,B,C are independent.

Define common information:

$$i(A;B) = i(A) + i(B) - i(AB)$$



Bayes' theorem: $i(A|B) = i(A) - i(A;B)$

Information and Propositions

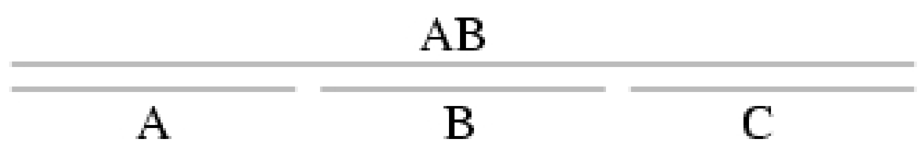
If $i(A;B) > 0$, then $i(A) + i(B) > i(AB)$

$\rightarrow$ can know ind. props. W, X, Y s.t. $A = WX$ and $B = XY$

$$i(A|B) = i(W)$$

If $i(A;B) < 0$, then $i(AB) > i(A) + i(B)$

$\rightarrow$ can know C s.t. $AB \Rightarrow C$ but $\neg(A \Rightarrow C)$ and $\neg(B \Rightarrow C)$.



E.g. $A =$ "All men are mortal"
$B =$ "Socrates is a man"
$C =$ "Socrates is mortal"

$$i(A|B) = i(A) + i(C)$$

If $P(A) + P(B) \leq 1$, then C can be independent of A and B.

Evidence and Information

$$e(A \to B) = i(A; B) - i(A; \bar{B}) = \log \frac{P(A|B)}{P(A|\bar{B})}$$

"The amount of evidence provided by A in favour of B"
= Contribution from A to the log of the odds of B:

$$\log \frac{P(B|A)}{P(\bar{B}|A)} = \log \frac{P(B)}{P(\bar{B})} + \log \frac{P(A|B)}{P(A|\bar{B})}$$

Independence conditions for evidence:

$$e(CD \to B) = e(C \to B) + e(D \to B)$$

$$\text{if} \quad i(C; D|B) = 0 \text{ and } i(C; D|\bar{B}) = 0$$

Independence conditions for information:

$$i(CD; B) = i(C; B) + i(D; B)$$

$$\text{if} \quad i(C; D|B) = 0 \text{ and } i(C; D) = 0$$

Example of Evidence and Information

Ann, Bob and David may or may not be in a particular house.

Propositions:
N: Nobody is in the house
($\equiv$ Ann is not in the house AND Bob is not...)

A: Ann is not in the house AND X
B: Bob is not in the house AND Y

C: There is no car at the house
L: The lights in the house are all off

A and B provide independent parts of the *information* required to believe N.
C and L provide independent *evidence* in favour of N.

Information $\rightarrow$ Proof
Evidence $\rightarrow$ Demonstration

Assumptions: Propositions asserting Ann, Bob and David's being in the house are independent of each other and of X and Y.
C and L independent given nobody in house and also given somebody in house.

Mutual Evidence

$w_{AB} = N_B$'s input if N_A spikes $-$ N_B's input if N_A doesn't spike

$$
\begin{aligned}
w_{AB} \text{ should } &= \text{(Evidence provided by a spike)} - \\
&\quad \text{(Evidence provided by absence of a spike)} \\
&= e(S_A \to S_B) - e(\overline{S_A} \to S_B) \\
&= i(S_A; S_B) - i(S_A; \overline{S_B}) - i(\overline{S_A}; S_B) + i(\overline{S_A}; \overline{S_B}) \\
&= e_m(S_A; S_B)
\end{aligned}
$$

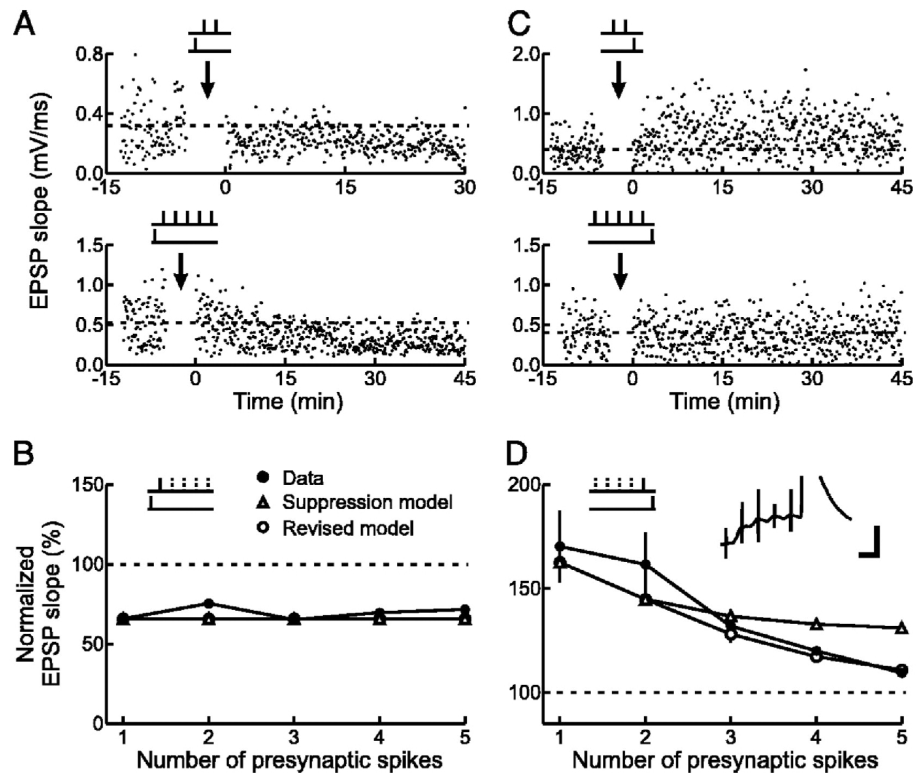
$$
w_{AB} \text{ should } = e_m(S_A; S_B) = \log \frac{P(\overline{S_A}\ \overline{S_B})P(S_A S_B)}{P(\overline{S_A} S_B)P(S_A \overline{S_B})}
$$

Learning rule:

and magnitude of pot./magnitude of dep. = $\exp(-w_{AB})$

Biological Data - Visual Cortex (Froemke et al, 2005)

Learning rules:



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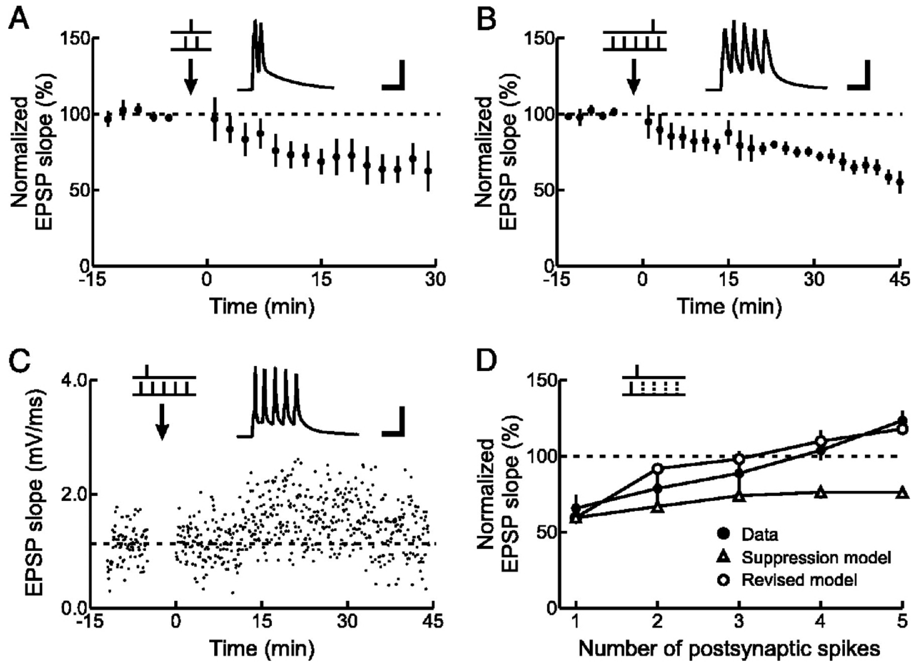


Legal Judgments

Judgment of Law: Made by a judge based on the information in accordance with the *letter of the law*. (Do the symbols match?)

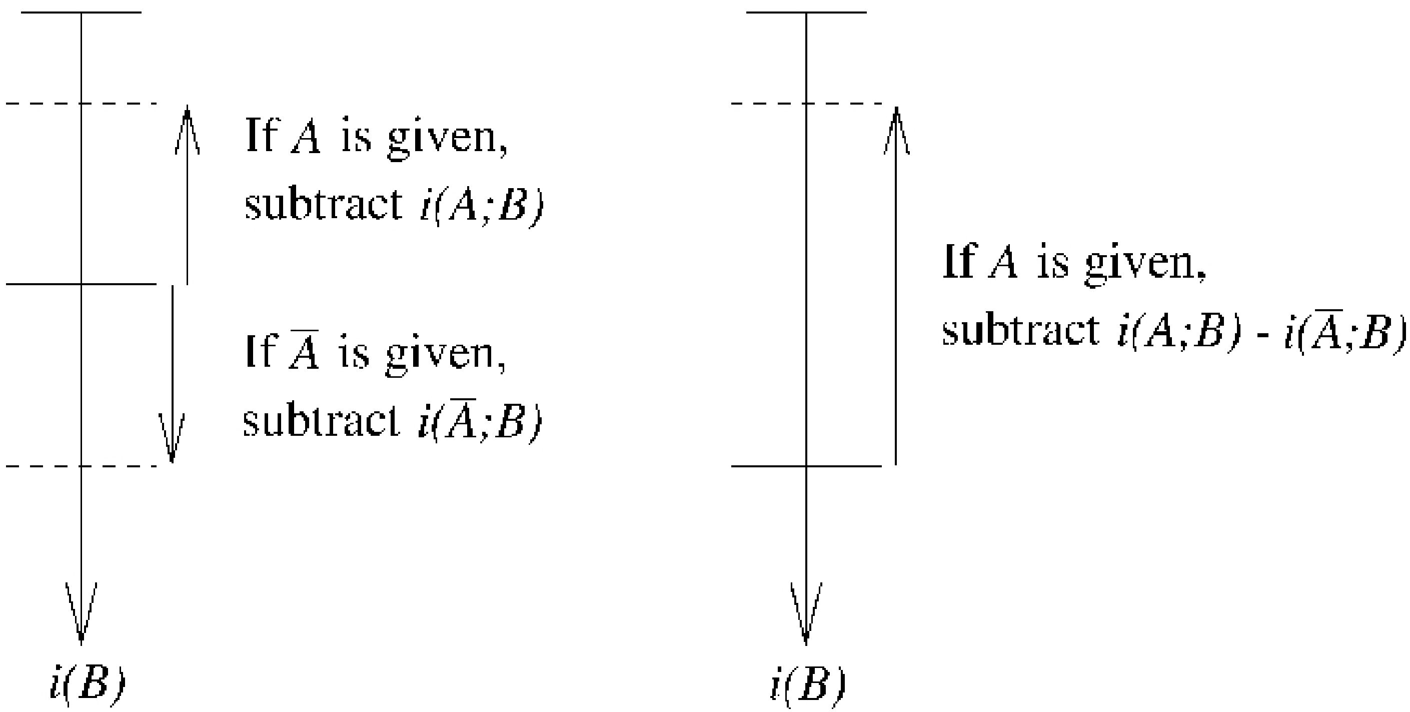
$\longrightarrow$ Symbolic reasoning, perfect proofs

Judgment of Fact: Made by a jury based on the evidence.

$\longrightarrow$ Hypothesis evaluation, evidence thresholds

Does $w_{AB} = e_m(S_A; S_B)$ mean that neurons perform hypothesis evaluation but not symbolic reasoning?

Two Strategies for Incorporating Information



"Speculate about A" "Assume A is false"

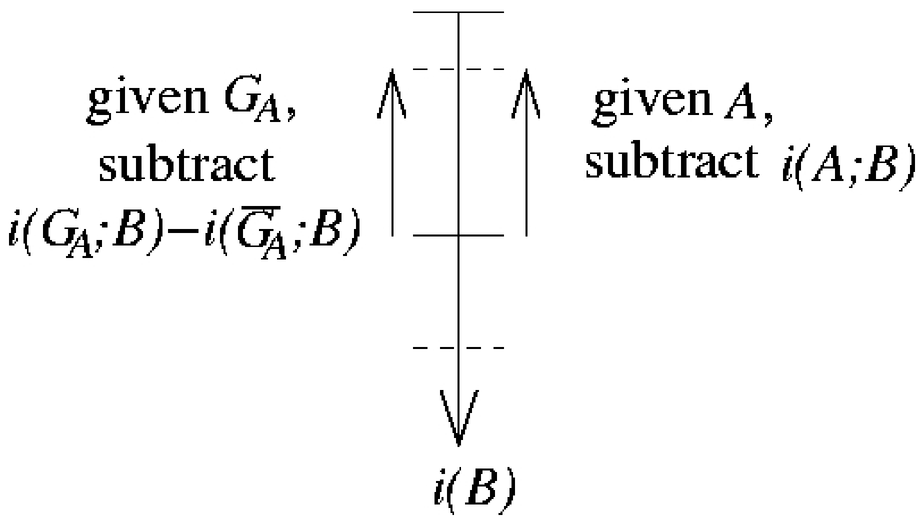
Can assume A is false if we are sure that A will be given if it is true. Assumption will be correct if A is not given.

The Proposition that A is Given

Never in any doubt about whether A is given.

$P(G_A) = 0$ or 1 $\qquad$ $P(A|G_A) = 1$

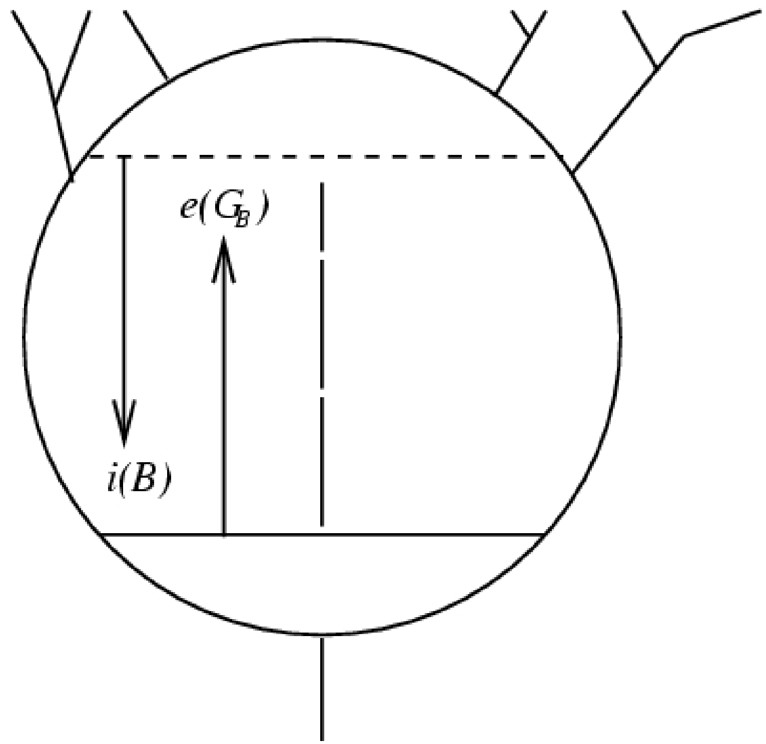
$$P(B|G_A) = P(B|A) \text{ unless } B = G_A$$



$$\begin{aligned}\Rightarrow i(A;B) &= i(G_A;B) - i(\overline{G_A};B) \\ &= i(G_A;G_B) - i(G_A;\overline{G_B}) - i(\overline{G_A};G_B) + i(\overline{G_A};\overline{G_B}) \\ &= e_m(G_A;G_B)\end{aligned}$$

$i(A;B) = e_m(G_A;G_B) \rightarrow$ "duality" between information and evidence.

$$w_{AB} = e_m(S_A; S_B) = e_m(G_A; G_B) = i(A; B)$$



Interpret spike as judgment $\rightarrow S_A = G_A$

$\Sigma_A e_m(G_A; G_B)$	$=$	$\Sigma_A i(A; B)$
G_B is demonstrated	$\Leftrightarrow$	B is proven
Hypothesis evaluation		Symbolic reasoning

Conclusion

Because $i(A;B) = e_m(G_A;G_B)$, integrate-and-fire neurons which use the learning rules:

can accomplish not only hypothesis evaluation on the basis of evidence but also symbolic reasoning on the basis of information.

Discrete Approximation

For each $t \in \{1, 2, 3, \cdots\}$

$$S_t = \sum_i w_i \chi_{it} \qquad \chi_{it} = \begin{cases} 1 & \text{if neuron } i \text{ spikes at t} \\ 0 & \text{otherwise} \end{cases}$$

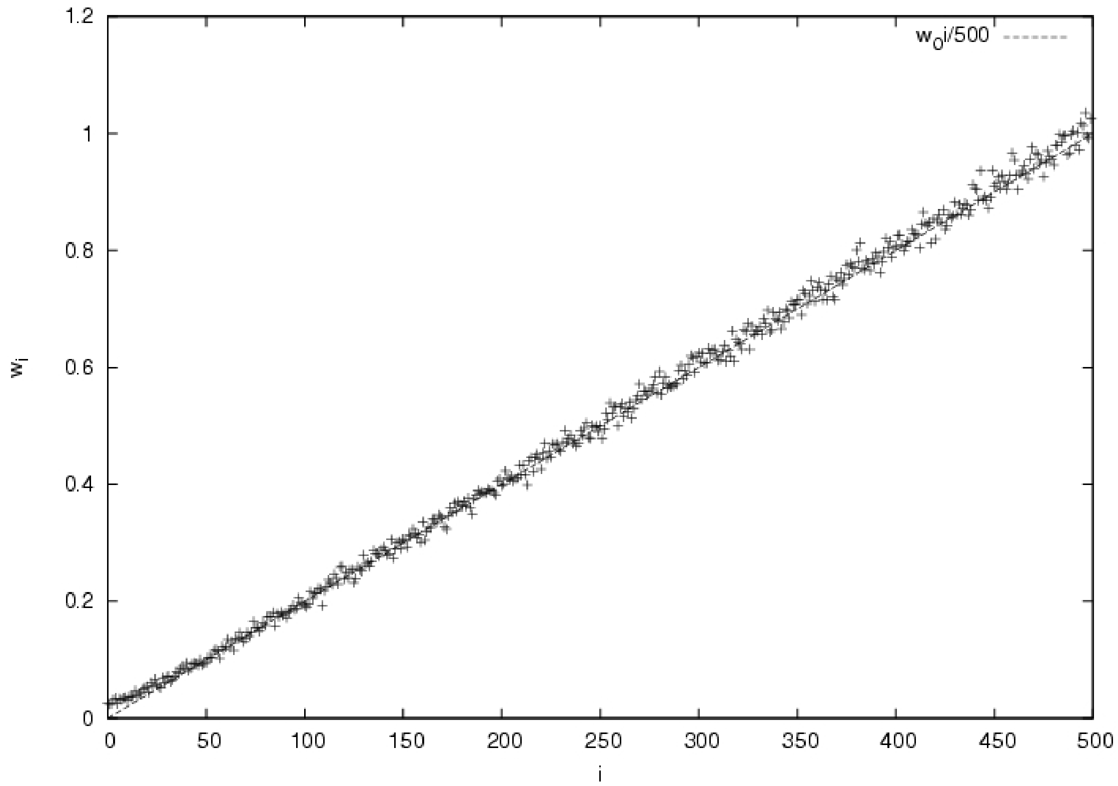
Postsynaptic neuron spikes at t if $S_t > V_{\text{th}}$.

Discrete version of STDP:

$w_i \to w_i(1 + ke^{-w_i})$ if
$\chi_{it} = 1$ and $S_t > V_{\text{th}}$ and $\chi_{i,t-1} = 0$ and $S_{t-1} < V_{\text{th}}$.

$w_i \to w_i(1 - k)$ if
$\chi_{i,t} = 0$ and $S_t > V_{\text{th}}$ and $\chi_{i,t+1} = 1$ and $S_{t+1} < V_{\text{th}}$.

Correlations



Simulation - 500 presynaptic neurons with firing correlated with event, E. $P(E) = 0.2$. Mutual evidence between proposition that E occurs and proposition that i fires was set to $i/500$.