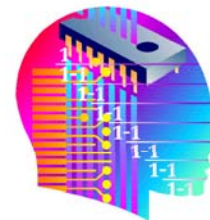


# Towards a Quantitative Framework for Sudden-Insight Problem Solving and Nonconscious Processing

Jerome Swartz, Robert DeBellis, Aileen Chou

Insights Into Insight 2008  
La Jolla, CA



# Acknowledgements

- Scott Makeig
- Nava Rubin
- Eugene Izhikevich
- Frank Hoppensteadt
- Jonathan Victor
- John Rinzel
- Hirsh Cohen

*many others...*

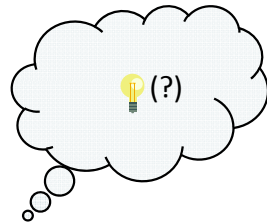
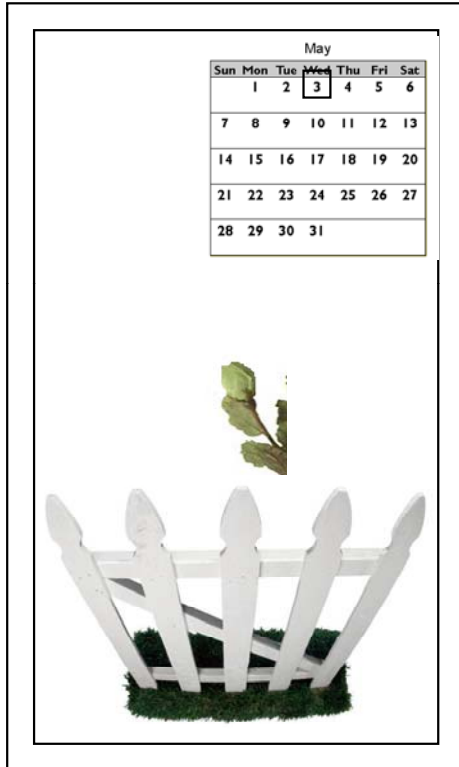
# Genesis

- Invention
- Creativity
- Puzzles
- Math Problems
- Abstract Reasoning

# The Rose in the Garden

“May Garden View” I

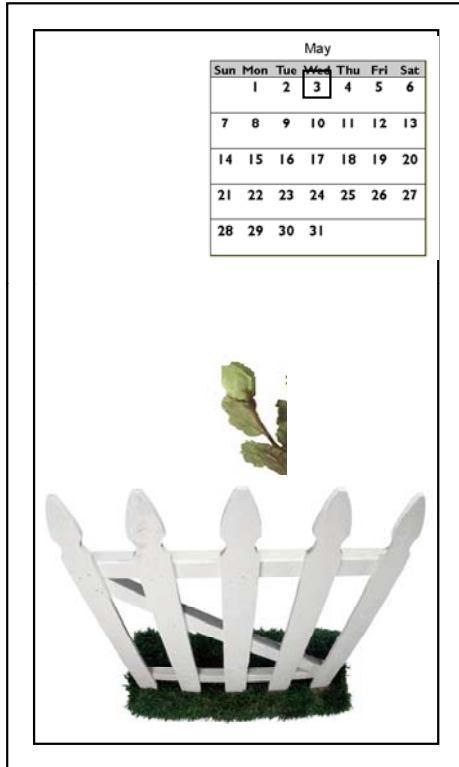
(e.g., e large)



# The Rose in the Garden

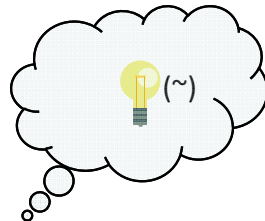
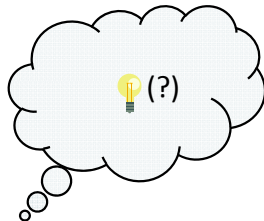
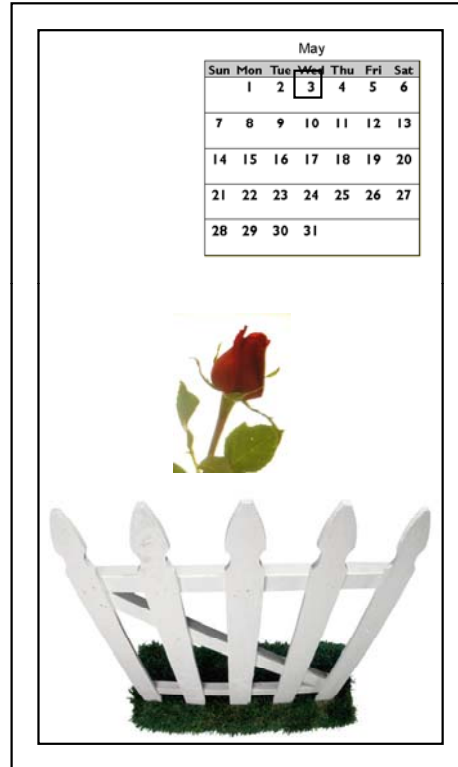
“May Garden View” I

(e.g., e large)



“May Garden View” II

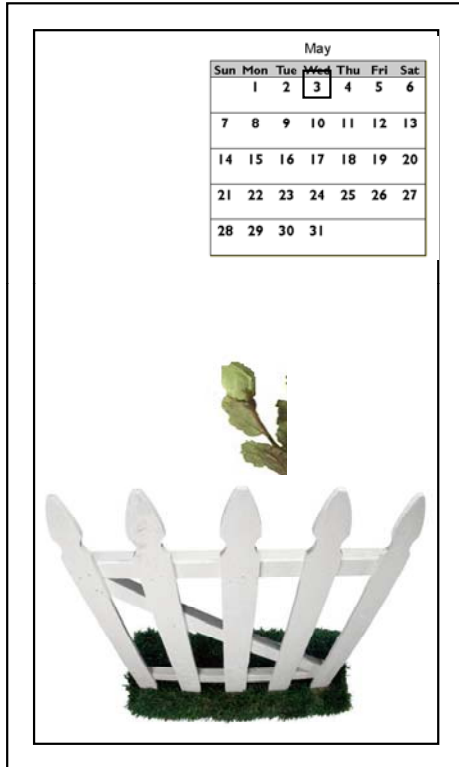
(e.g., e medium)



# The Rose in the Garden

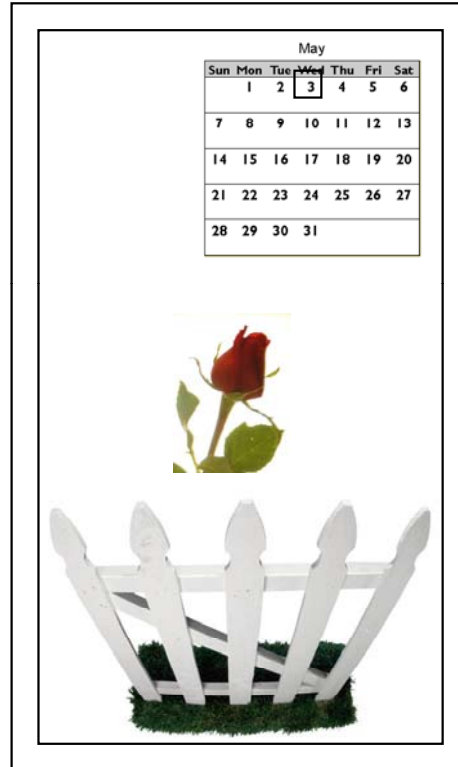
“May Garden View” I

(e.g., e large)



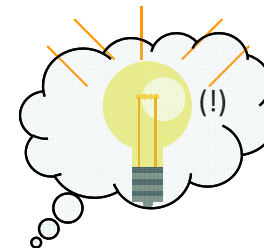
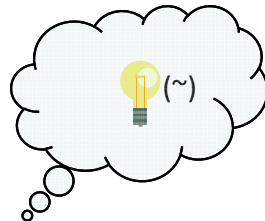
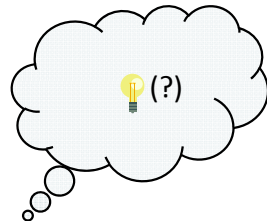
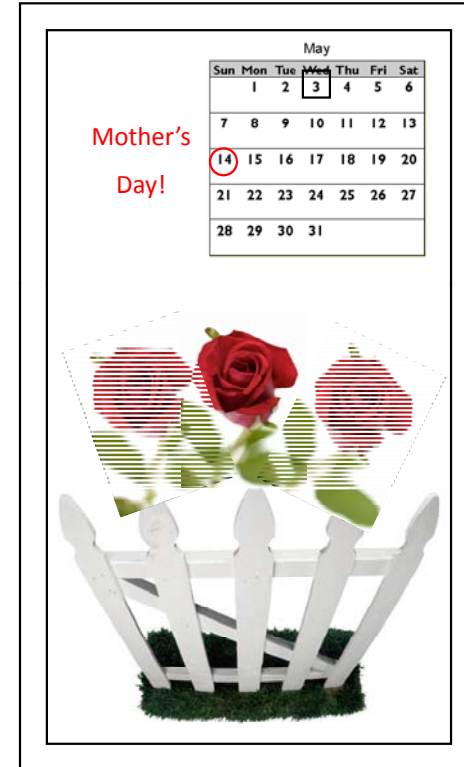
“May Garden View” II

(e.g., e medium)

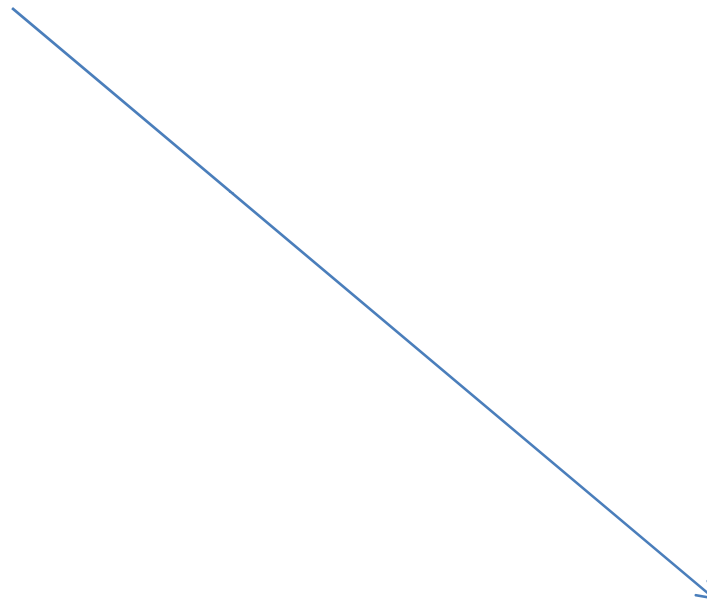
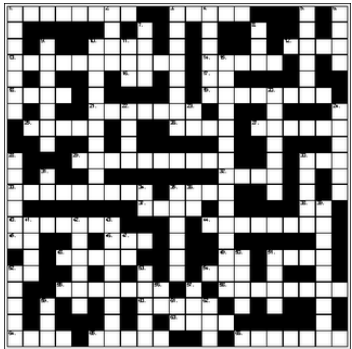


“May Garden View” III

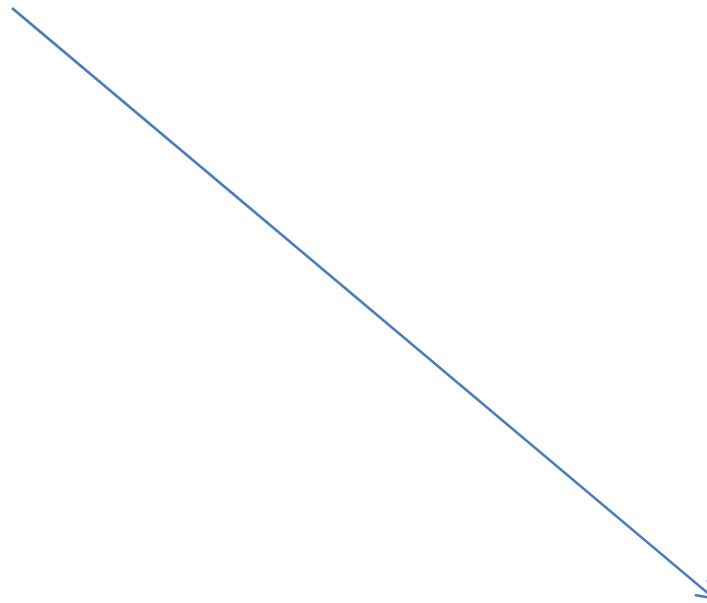
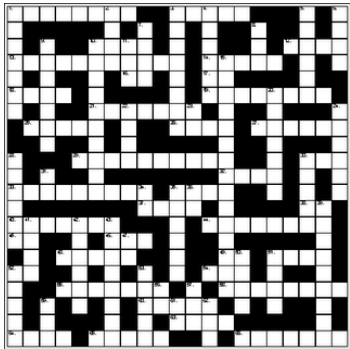
(e.g., e -> 0)



# “Standard-cut vegetable (7)”



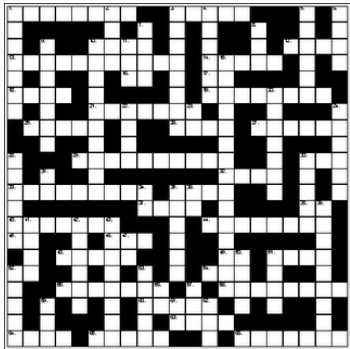
# “Standard-cut vegetable (7)”



+



# “Standard-cut vegetable (7)”



+



Parsnip

# 'Aha!'

“...the **clearest defining characteristic** of insight problem solving is the subjective “Aha!” or “Eureka!” experience that follows insight solutions...”

-- Neural Activity When People Solve Verbal Problems with Insight

**Mark Jung-Beeman, Edward M. Bowden, Jason Haberman, Jennifer L. Frymiare, Stella Arambel-Liu, Richard Greenblatt, Paul J. Reber, John Kounios**

PLoS Biology, 4/04

# Problem Solving

- Difficulty
- Representation
- Complexity
- Insight vs. Gradual
- LTM/priming
- 'Aha!' (vs. Duh!)

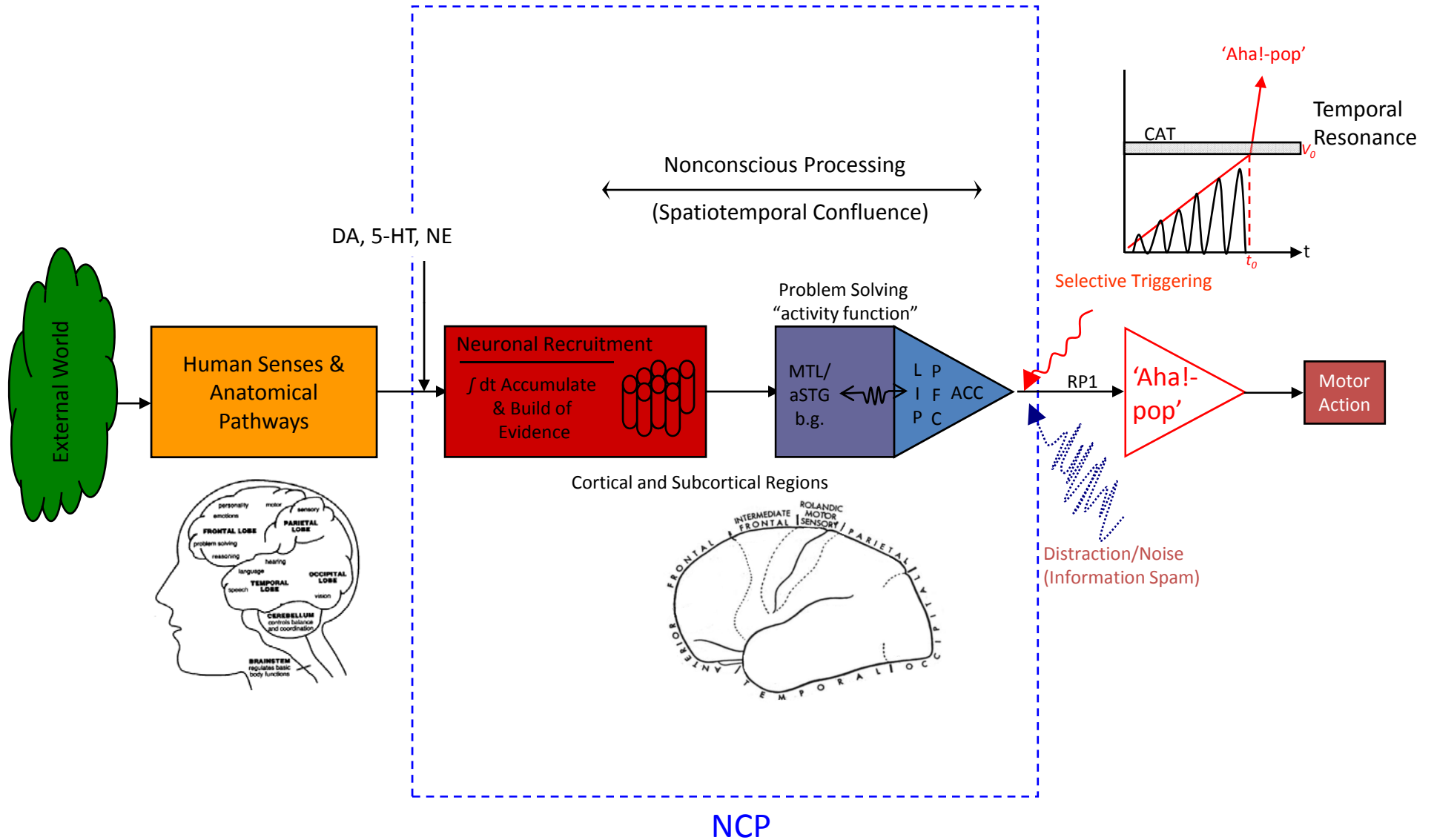


# 'Aha!' *A priori*

- Impasse
- All-or-none solution, but...not necessarily correct
- Emotional response
- Accumulation process (time), multiple areas (space)
- Motivation and/or incentive
- Noise dependence
- Invariance

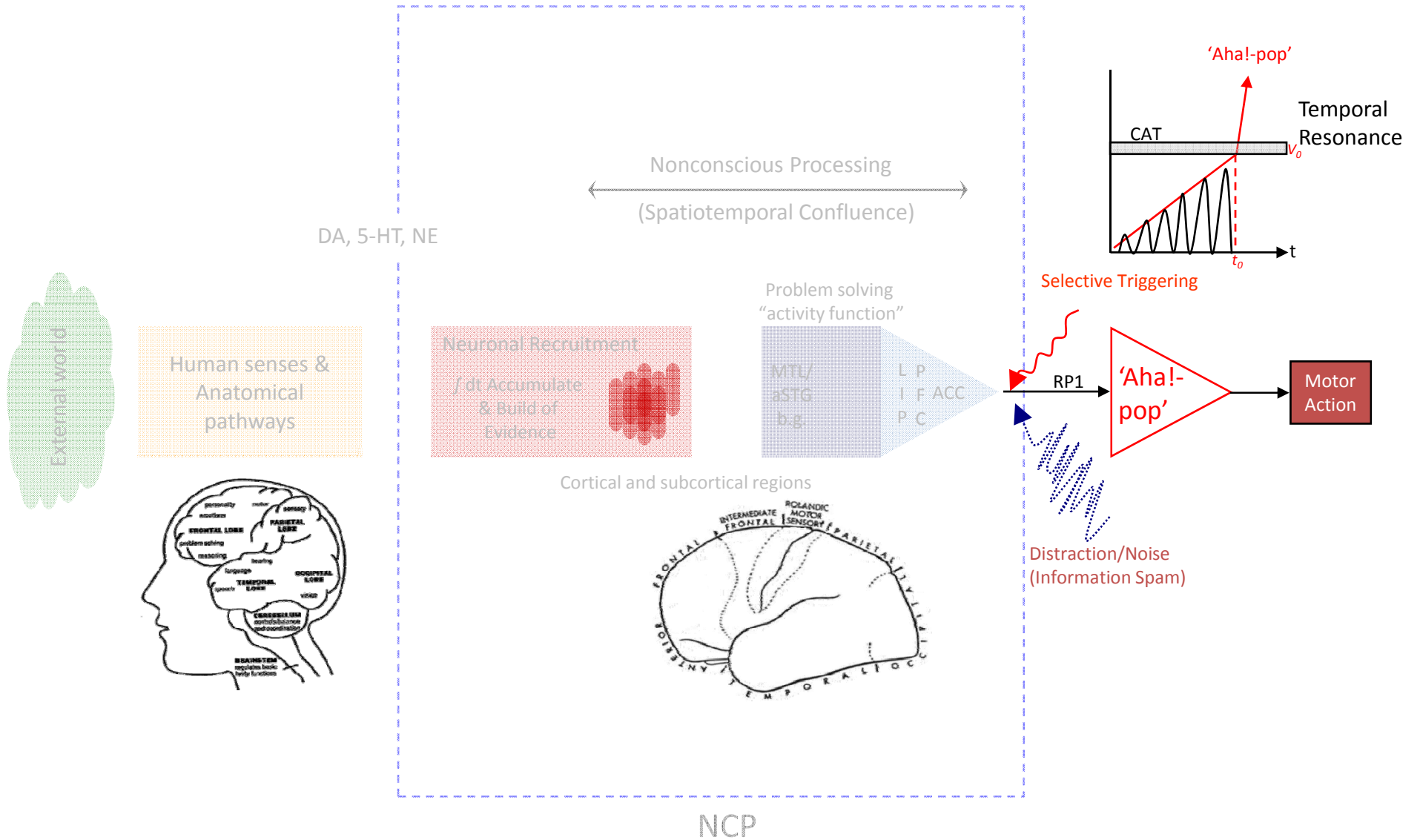
# Generalized NCP/ 'Aha!-pop' Conceptual Model

(Flow Diagram)



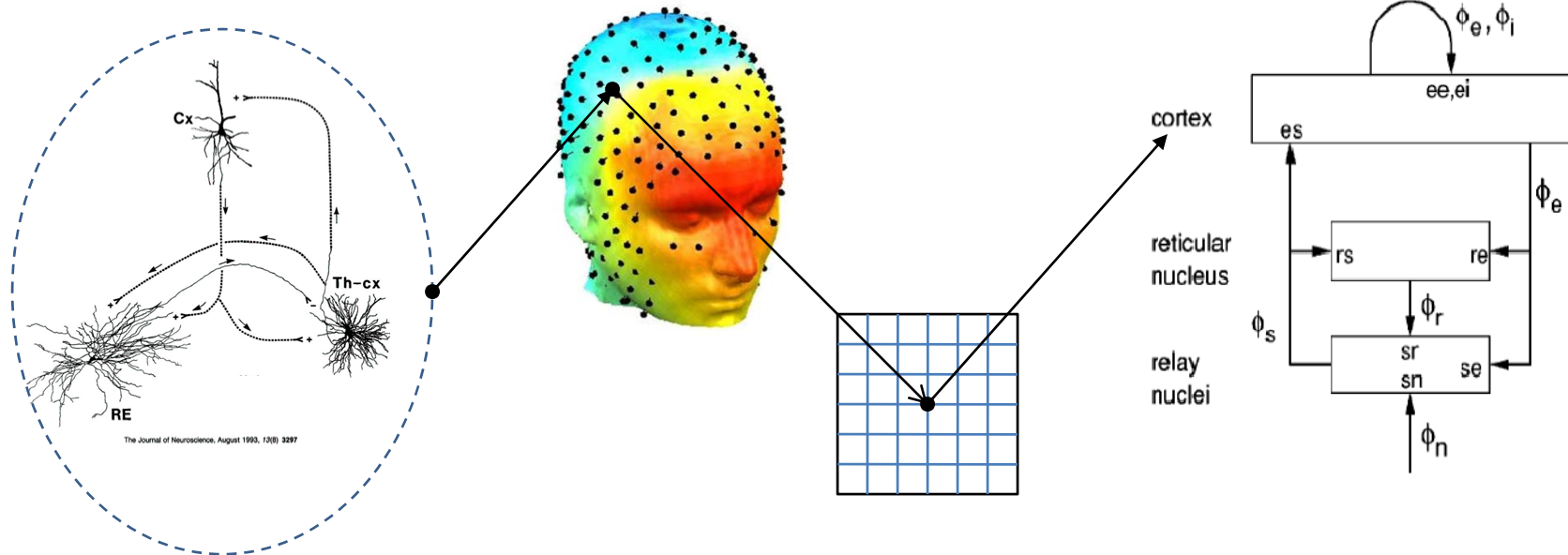
# Generalized NCP/ 'Aha!-pop' Conceptual Model

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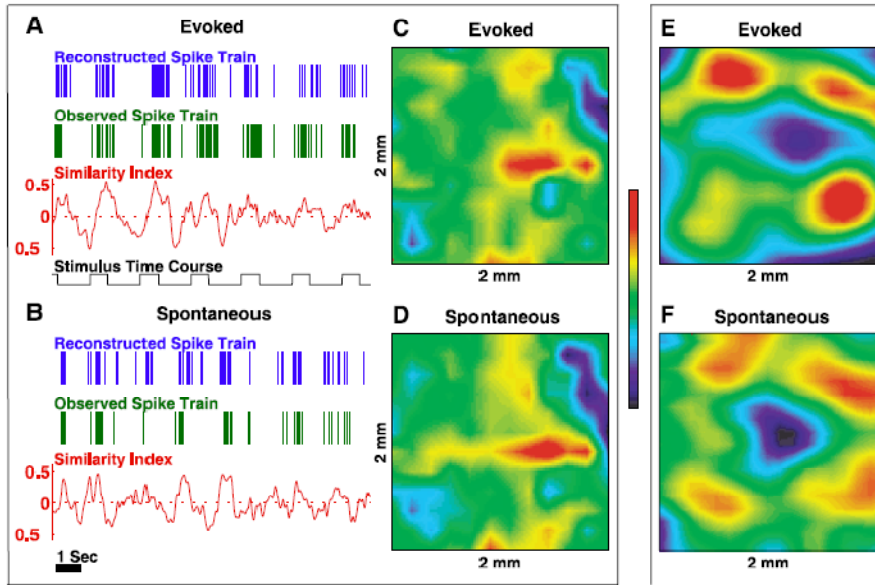
# Choosing a Model

- Mean-field
- Specifically models EEG/ECoG
- 'Realistic'
- Extensible
- Linearity still meaningful
- Predictor



# Mean Field Approaches

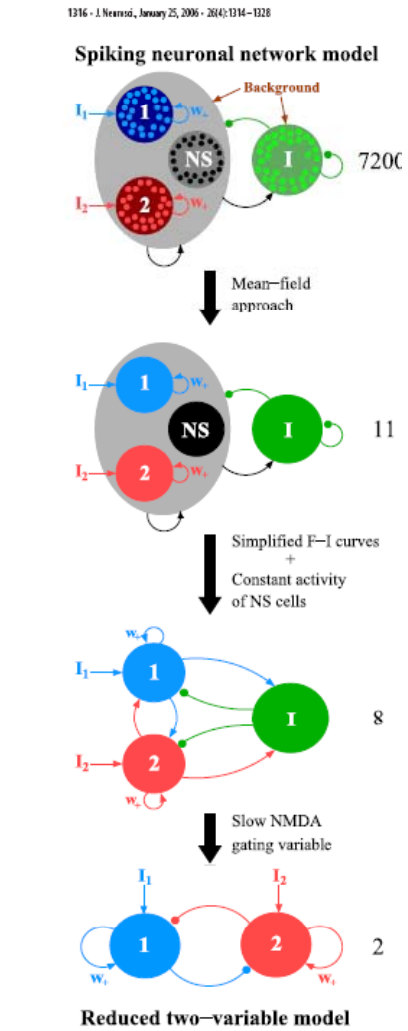
## Biological $\longleftrightarrow$ Computational



*In vivo*

Tsodyks, et al. 1998

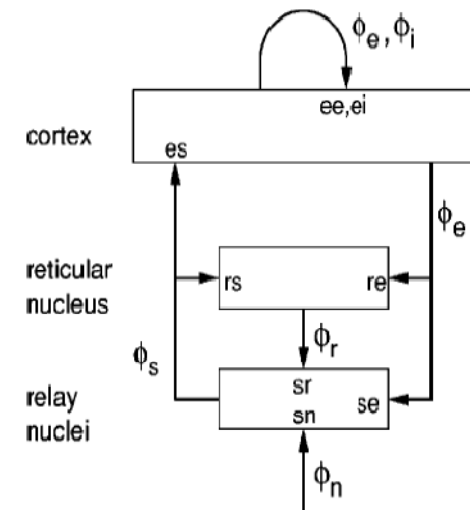
Wong and Wang, 2006



*In silica*

# Robinson's Cortico-Thalamocortical Model

- Specifically models EEG/ECoG
- Delay-Dependent (DDE) and linearized PDE  $\rightarrow$  ODE
- 18 physiologically-based, experimentally-verified parameters
- Parametric decoupling of compartments
- Extensible – more complicated configurations may be constructed across distinct brain regions
- Captures many EEG-specific features in linear dynamics (Breakspear, DeBellis)



*A linear resonator model is the simplest instantiation for a stimulus-triggered rise-to-threshold*

# Robinson Model of Neural Population Dynamics

(Historical development via Wilson & Cowan, Freeman,  
Amari, Haken, Nunez, and Wright & Liley)

Nonlinear form (Robinson et al., 1997)

$$V_a(t) = \int_{-\infty}^{\infty} L(t-t')P_a(t')dt'$$

$$P_a(t) = \sum_b v_{ab}\phi_b(t)$$

$$\left( \frac{1}{\gamma_a^2} \frac{\partial^2}{\partial t^2} + \frac{2}{\gamma_a} \frac{\partial}{\partial t} + 1 - r_a^2 \nabla^2 \right) \phi_a(\mathbf{r}, t) = Q(V_a(\mathbf{r}, t))$$

$$L(u) = \frac{\alpha\beta}{\beta - \alpha} (e^{-\alpha u} - e^{-\beta u})$$

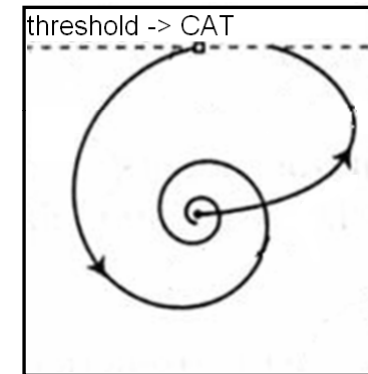
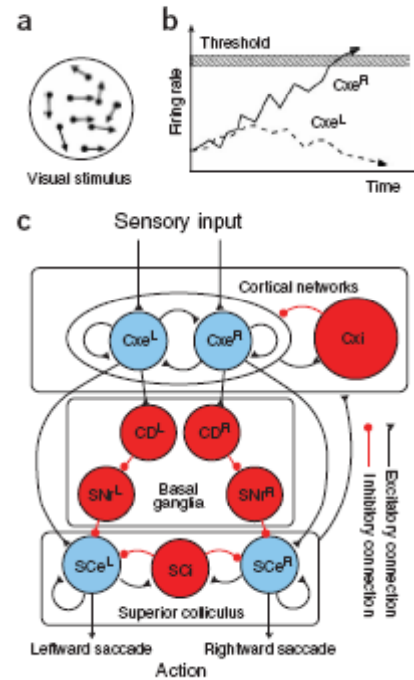
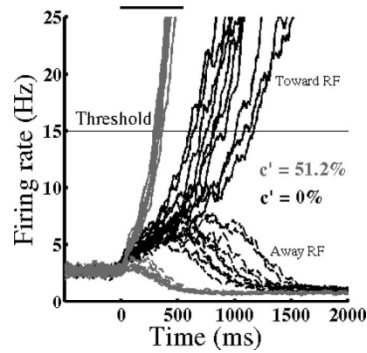
$$Q(x) = \frac{Q_{\max}}{1 + e^{\frac{-(V(x)-\theta)}{\sigma}}}$$



Space-clamped, Linearized form (Robinson et al., 2005)

$$D_\alpha V_a(\mathbf{r}, t) = \sum_b N_{ab} s_{ab} \phi_b(\mathbf{r}, t - \tau_{ab}) \quad D_\alpha = \frac{1}{\alpha\beta} \frac{d^2}{dt^2} + \left( \frac{1}{\alpha} + \frac{1}{\beta} \right) \frac{d}{dt} + 1$$

# Thresholds and Bifurcations



# Fuzzy Pole Math

$$D_\alpha V_a(\mathbf{r}, t) = \sum_b N_{ab} s_{ab} \phi_b(\mathbf{r}, t - \tau_{ab}) \quad D_\alpha = \frac{1}{\alpha\beta} \frac{d^2}{dt^2} + \left( \frac{1}{\alpha} + \frac{1}{\beta} \right) \frac{d}{dt} + 1$$

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In time domain, we reduce the Robinson ODE to

$$\frac{d^2 V_a}{dt^2} + (\alpha + \beta) \frac{dV_a}{dt} + \alpha\beta V_a = A\alpha\beta e^{-\gamma t} + \dots$$

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The Laplace-transformed equation is

$$(s + \alpha)(s + \beta)V_a(s) = \frac{A\alpha\beta}{(s + \gamma)} + \dots$$

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For  $\gamma = \alpha$  (double pole), the  $Ae^{-\gamma t}$  RHS forcing function yields a steady state solution with a **Real part**:

$$V_a(t) = \frac{A\alpha\beta}{\beta - \alpha} t \cos \alpha t$$

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The near-resonant “fuzzy-pole” expansion for  $\beta$  or  $\gamma = \alpha + \varepsilon$  ( $\varepsilon$  small), yields a transfer function kernel

$$H_0(s) = \frac{1}{(s + \alpha)(s + \alpha + \varepsilon)} \approx \frac{1}{(s + \alpha)^2} \left[ 1 - \frac{\varepsilon}{(s + \alpha)} + \frac{\varepsilon^2}{(s + \alpha)^2} - \dots \right]$$

# Fuzzy Pole Math

$$D_\alpha V_a(\mathbf{r}, t) = \sum_b N_{ab} s_{ab} \phi_b(\mathbf{r}, t - \tau_{ab}) \quad D_\alpha = \frac{1}{\alpha\beta} \frac{d^2}{dt^2} + \left( \frac{1}{\alpha} + \frac{1}{\beta} \right) \frac{d}{dt} + 1$$

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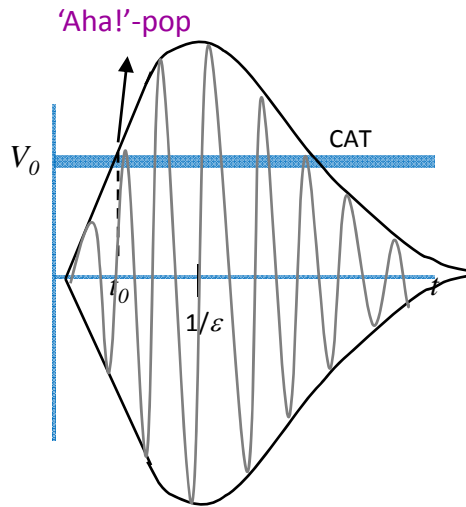
The real part of the inverse transform of the above equation is then of the form

$$h(t) \approx t \cos \alpha t \left( 1 - \frac{\varepsilon t}{2} + \frac{(\varepsilon t)^2}{6} - \dots \right), \text{ so that } V_a(t) = \frac{A\alpha\beta}{\beta - \alpha} t \cos \alpha t \left[ 1 - \frac{\varepsilon t}{2} + \frac{(\varepsilon t)^2}{6} - \dots \right]$$

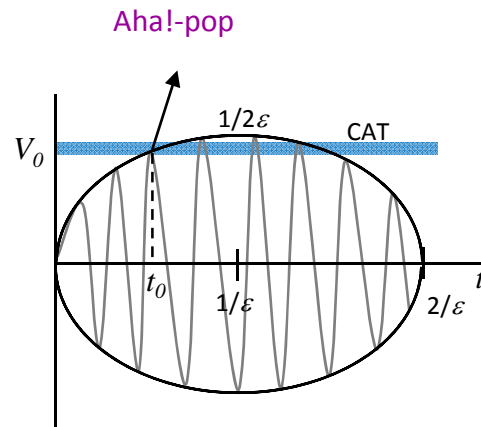
# Resonance and “Conscious Access”

$$V_a(t) \approx \frac{A\alpha\beta}{\beta - \alpha} t \cos \alpha t \left[ 1 - \frac{\varepsilon t}{2} \right]$$

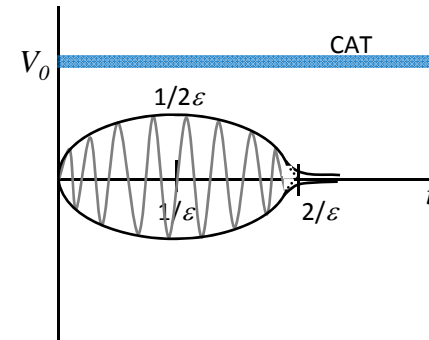
$\gamma = \alpha ; \varepsilon = 0$   
(linear ramp-to-threshold)



$\gamma = \alpha + \varepsilon, \varepsilon$  small  
(threshold crossing)



$\gamma = \alpha + \varepsilon, \varepsilon$  large  
(no threshold crossing)

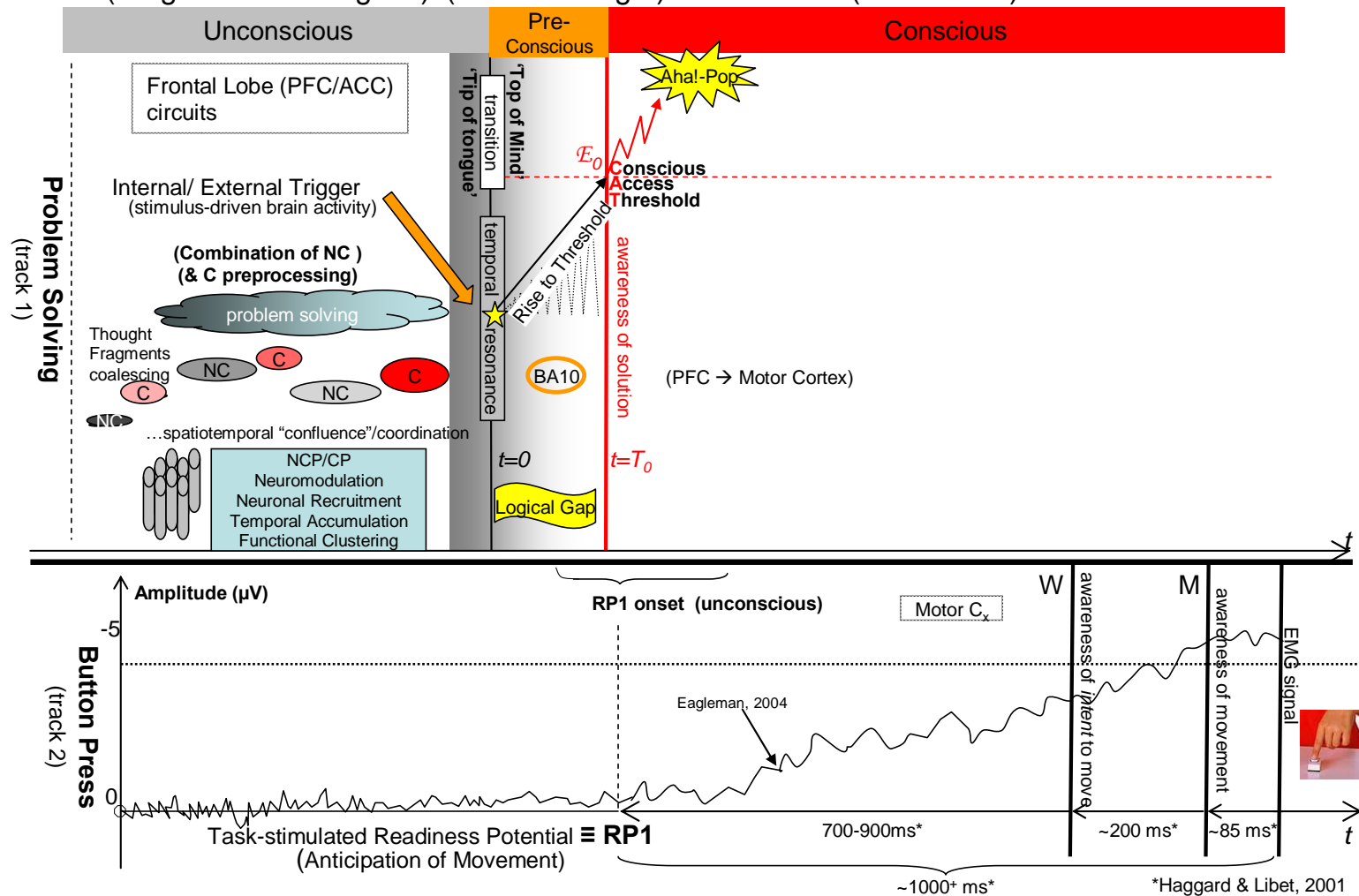


# Predictions from Model

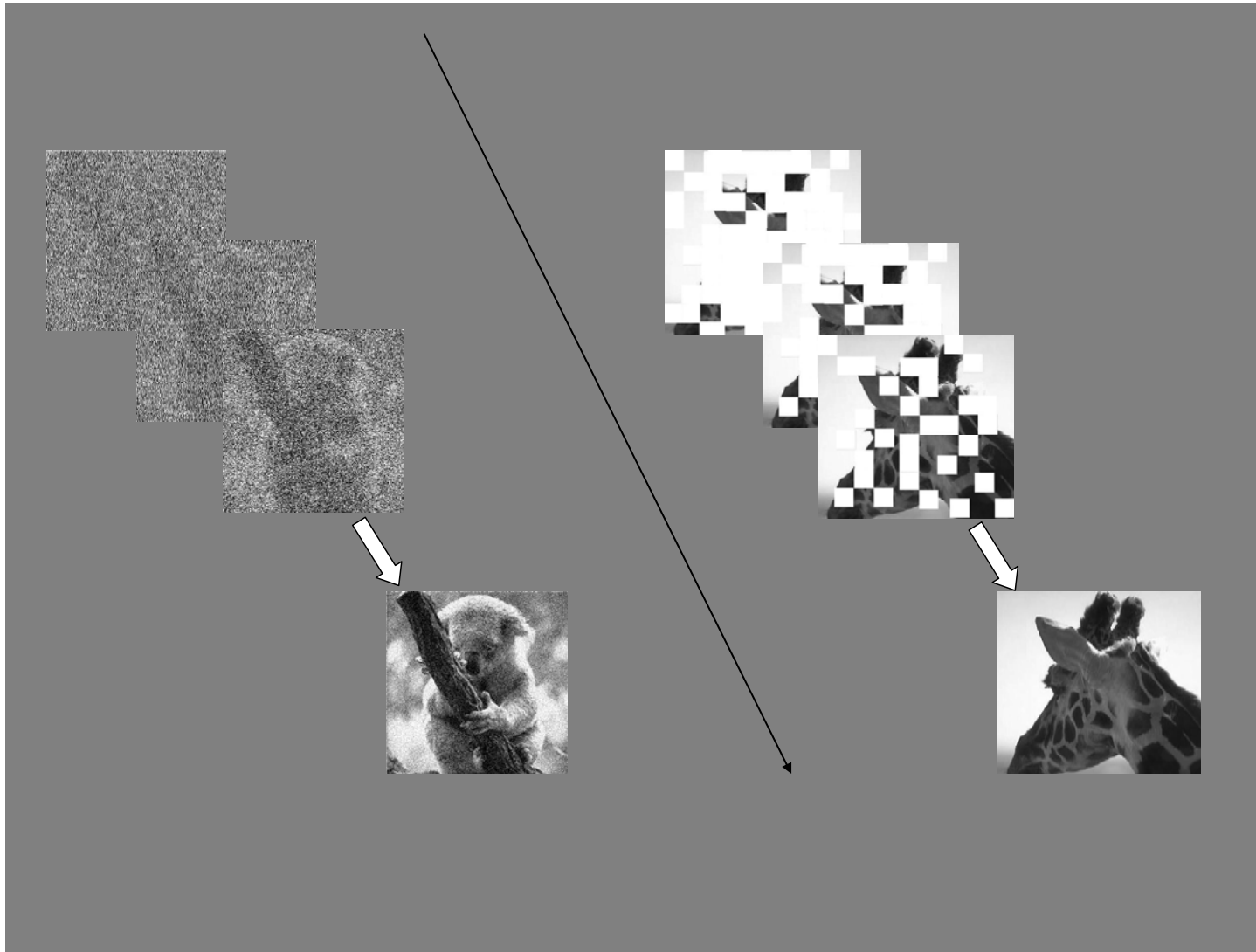
- 'Aha!' generates EEG activity with envelopes with linear-exponential envelopes
- EEG gamma band peaks at  $t_0$
- Stimulus 'closeness' represented by  $\varepsilon$  parameterizes EEG response
- 'Distraction/Noise' attenuates response

# Interleaving of Time Courses

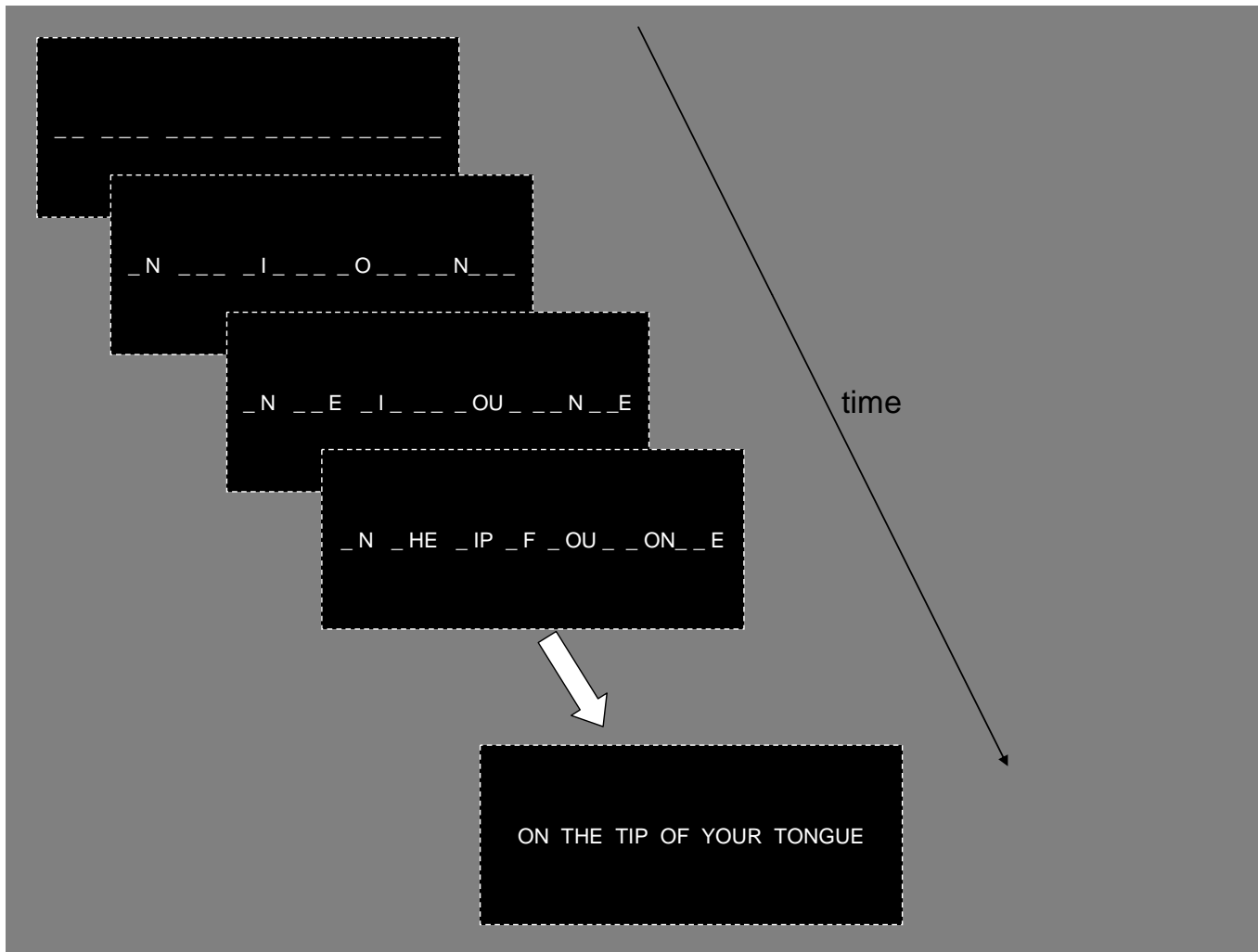
Time Courses of Insight Problem Solving & Button Press Dynamics (Conceptual Model)  
 ... Relating 'NCP/Aha!-pop' with Readiness Potential & Reports of Awareness  
 (Fragmented Thoughts) (Sudden Insight) (Awareness)



# Camouflage Reveal



# Phrase Completion



# Next Steps

NYU – camouflaged image psychophysics (with N. Rubin)

NYU – computational modeling (with F. Hoppensteadt and J. Rinzel)

UCSD – EEG/ICA dynamics (with S. Makeig)

Thank you