Towards a Quantitative Framework for Sudden-Insight Problem Solving and Nonconscious Processing

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Acknowledgements

- Scott Makeig
- Nava Rubin
- Eugene Izhikevich
- Frank Hoppensteadt
- Jonathan Victor
- John Rinzel
- Hirsh Cohen

many others…

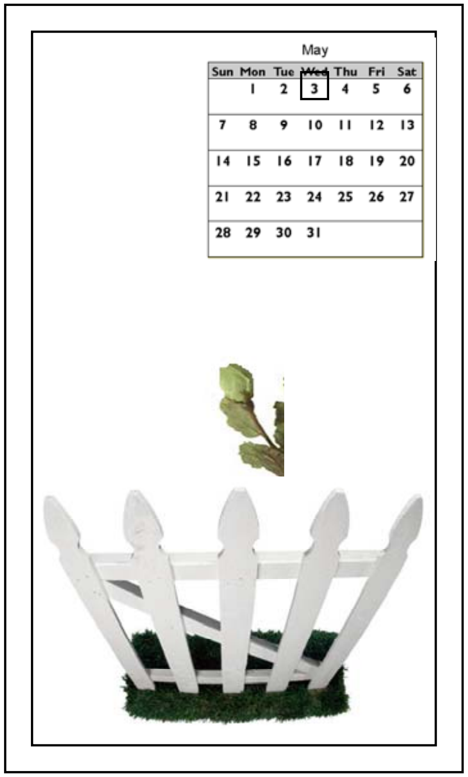
Genesis

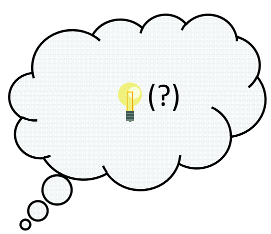
- Invention
- Creativity
- Puzzles
- Math Problems
- Abstract Reasoning

The Rose in the Garden

"May Garden View" I

(e.g., e large)





The Rose in the Garden

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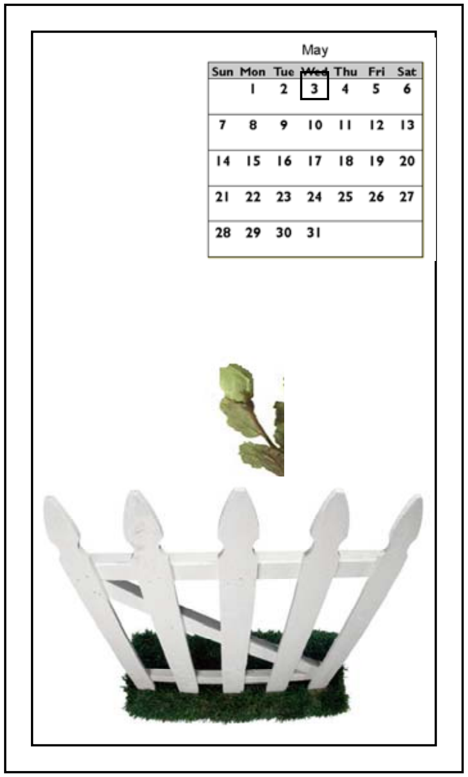
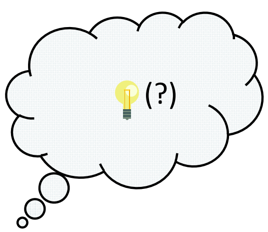
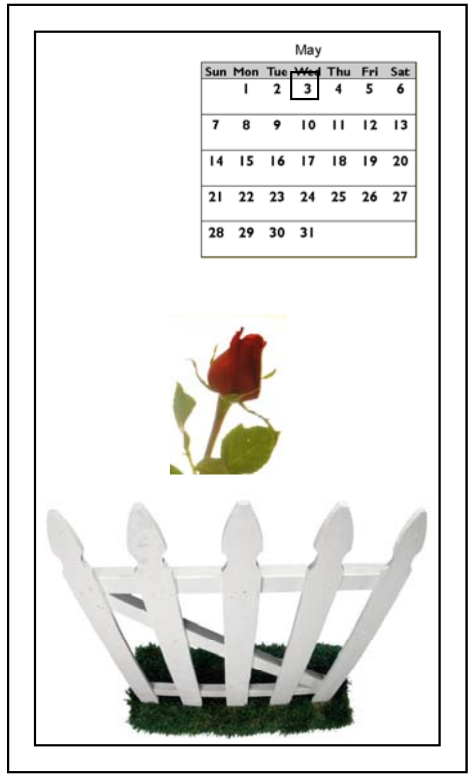
(e.g., e large)

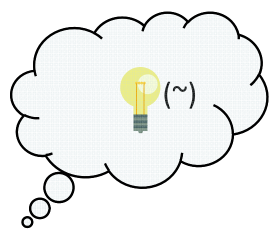




“May Garden View” II

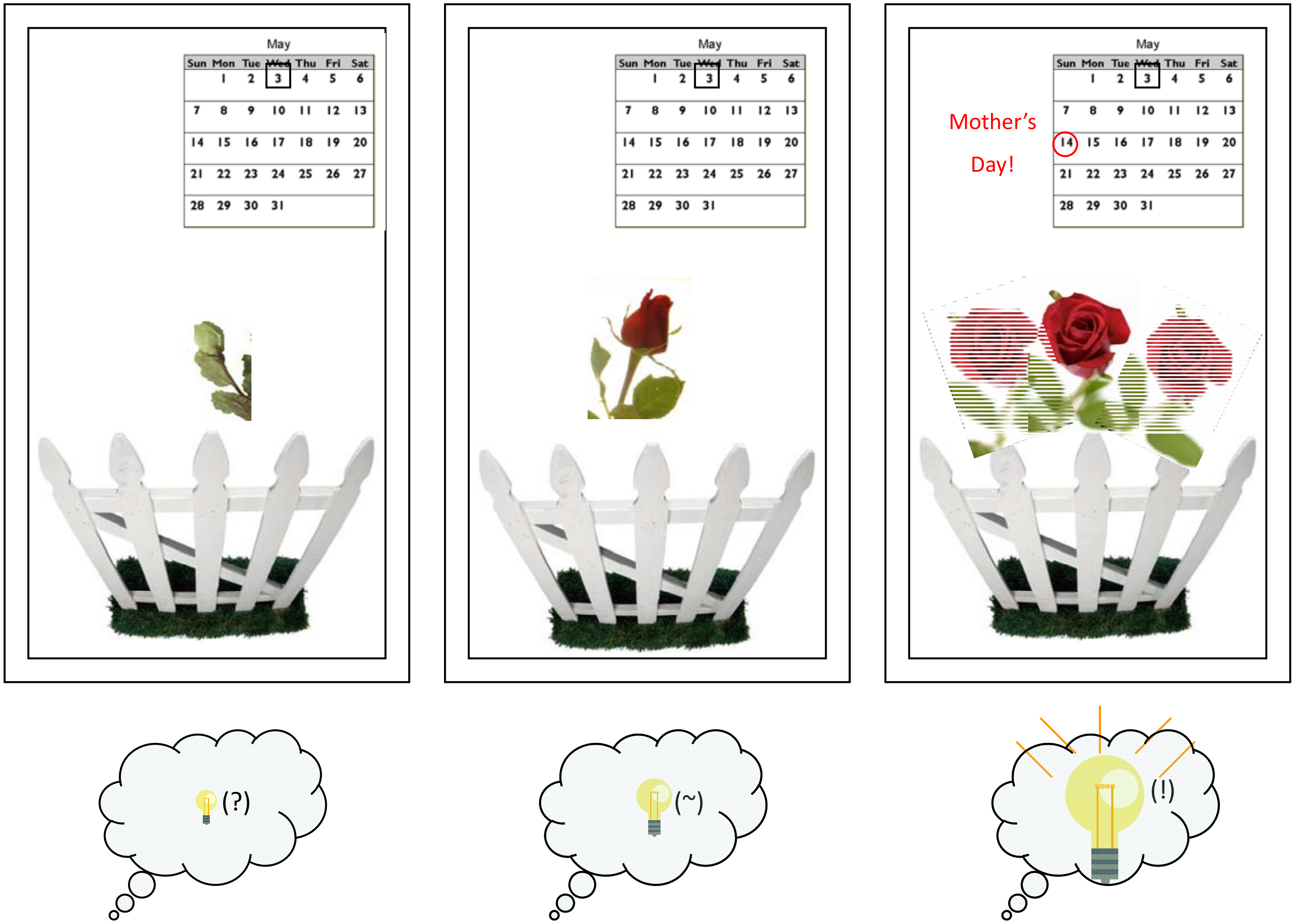
(e.g., e medium)



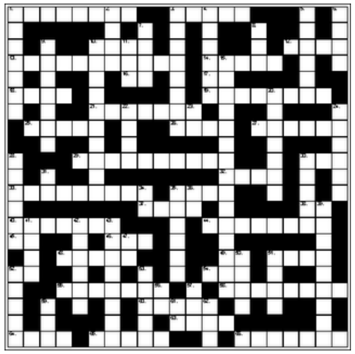


The Rose in the Garden

“May Garden View” I	“May Garden View” II	“May Garden View” III
(e.g., e large)	*(e.g., e medium)*	*(e.g., e -> 0)*



“Standard-cut vegetable (7)”



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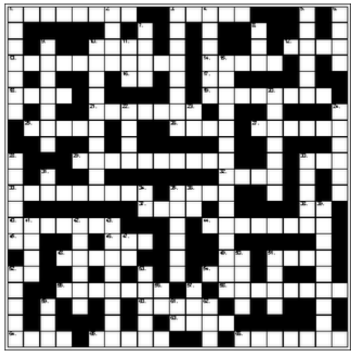

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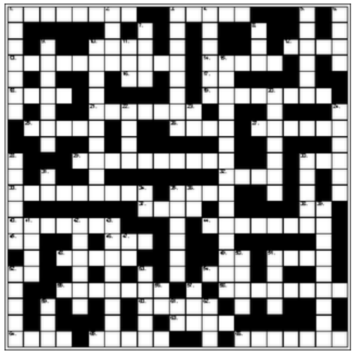

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Parsnip

‘Aha!’

“…the clearest defining characteristic of insight problem solving is the subjective ‘‘Aha!’’ or ‘‘Eureka!’’ experience that follows insight solutions…”

-- Neural Activity When People Solve Verbal Problems with Insight

Mark Jung-Beeman, Edward M. Bowden, Jason Haberman, Jennifer L. Frymiare, Stella Arambel-Liu, Richard Greenblatt, Paul J. Reber, John Kounios

PLoS Biology, 4/04

Problem Solving

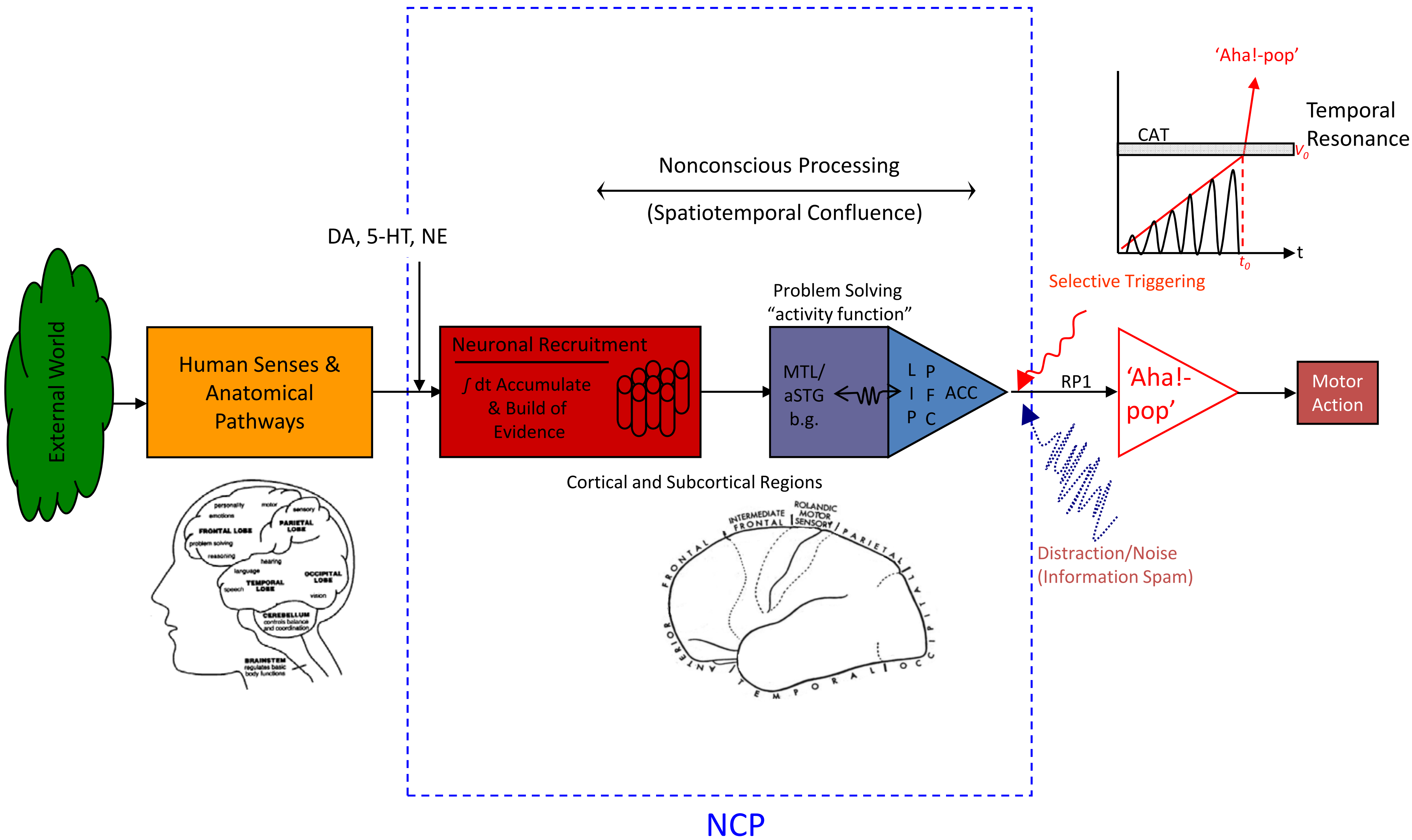
- Difficulty
- Representation
- Complexity
- Insight vs. Gradual
- LTM/priming
- 'Aha!' (vs. Duh!)

‘Aha!’ *A priori*

- Impasse
- All-or-none solution, but…not necessarily correct
- Emotional response
- Accumulation process (time), multiple areas (space)
- Motivation and/or incentive
- Noise dependence
- Invariance

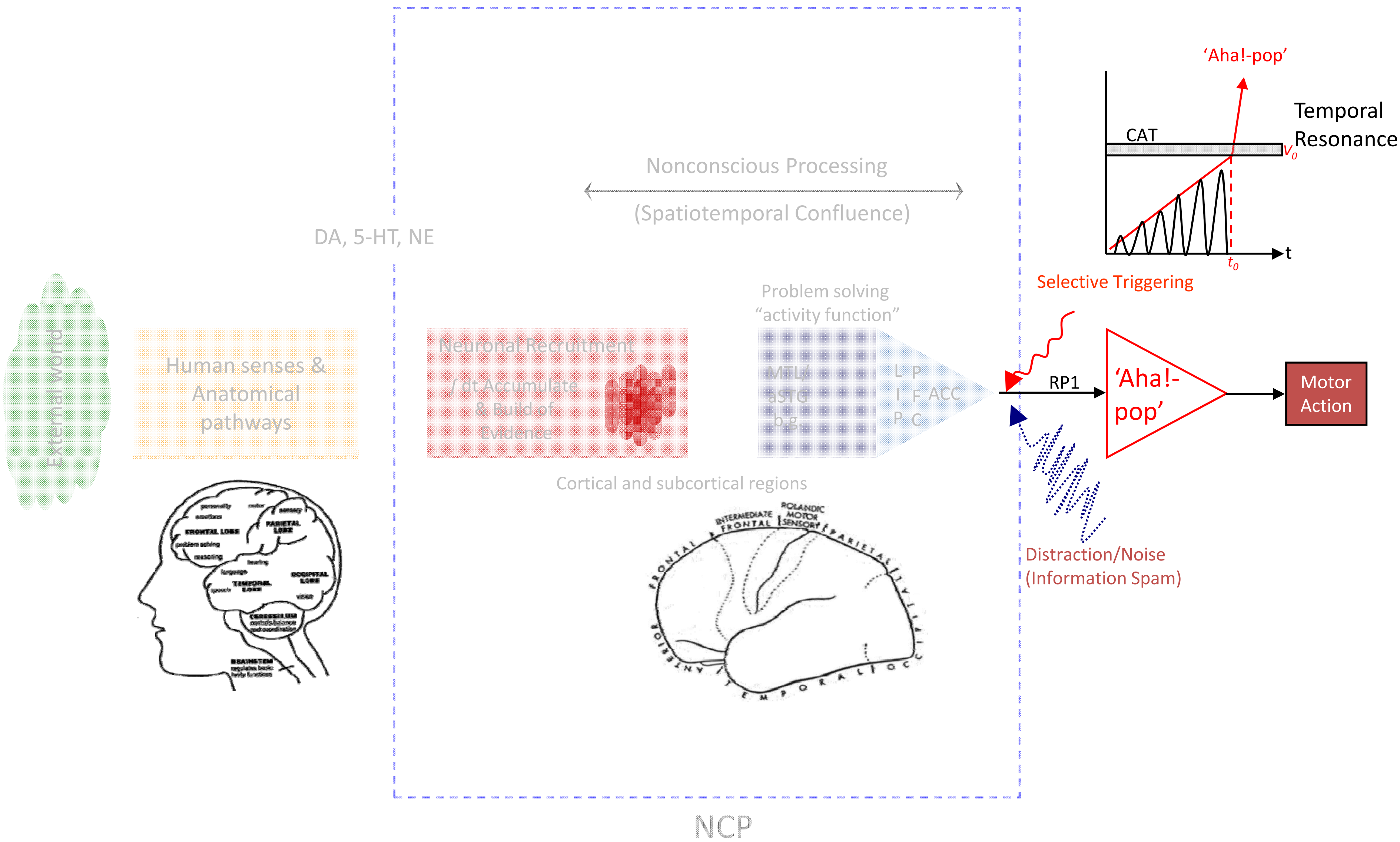
Generalized NCP/ ‘Aha!-pop’ Conceptual Model

(*Flow Diagram*)



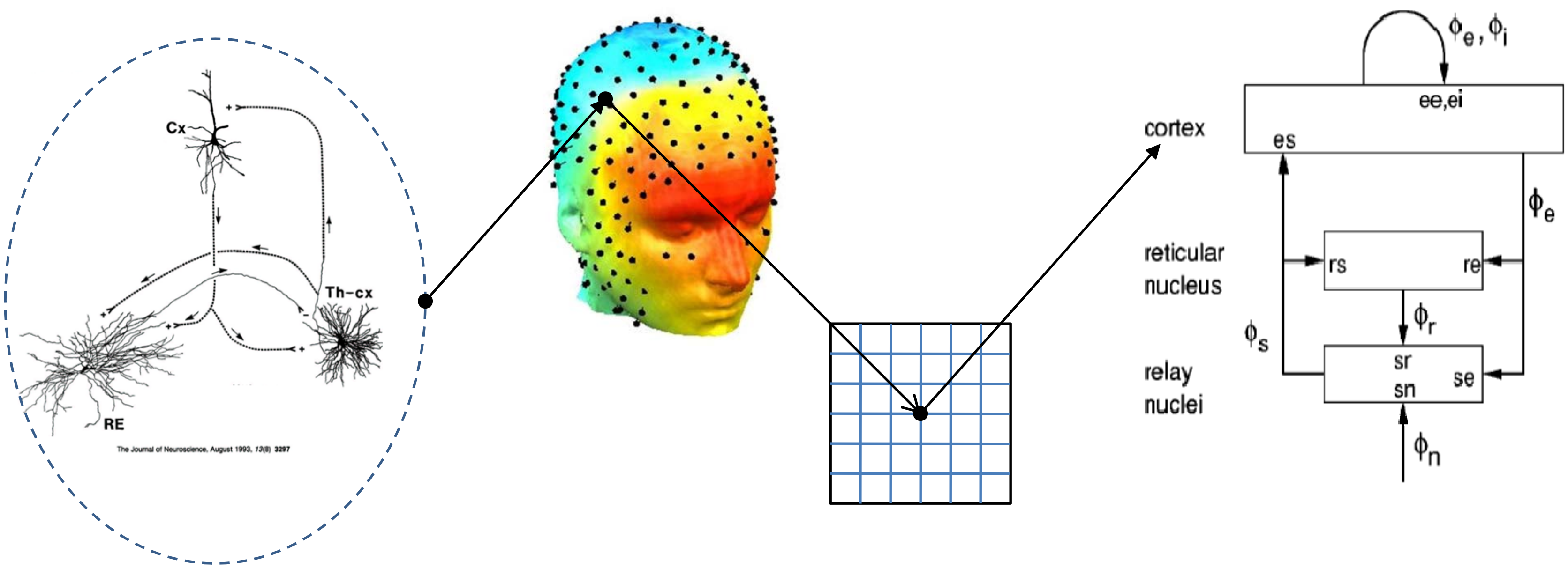
Generalized NCP/ 'Aha!-pop' Conceptual Model

(*Flow Diagram*)



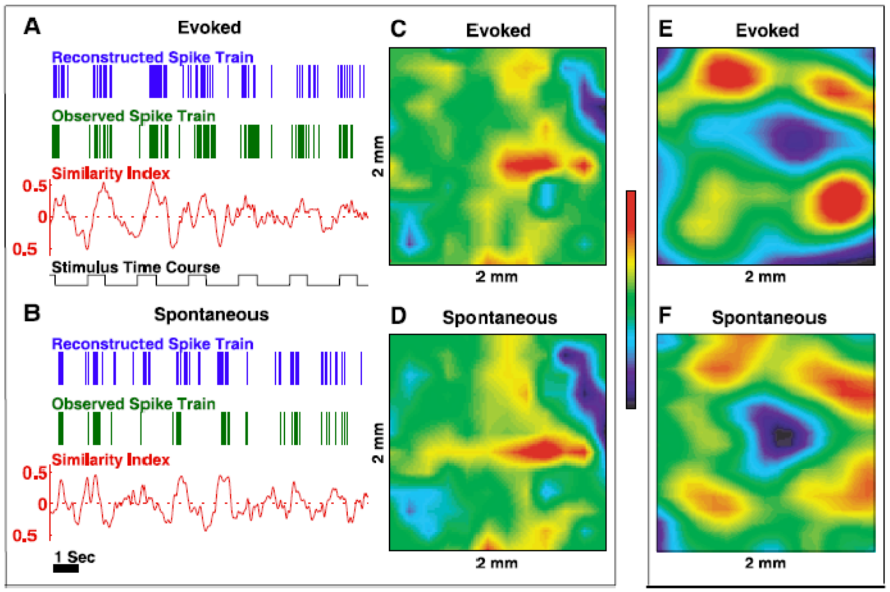
Choosing a Model

- Mean-field
- Specifically models EEG/ECoG
- ‘Realistic’
- Extensible
- Linearity still meaningful
- Predictor



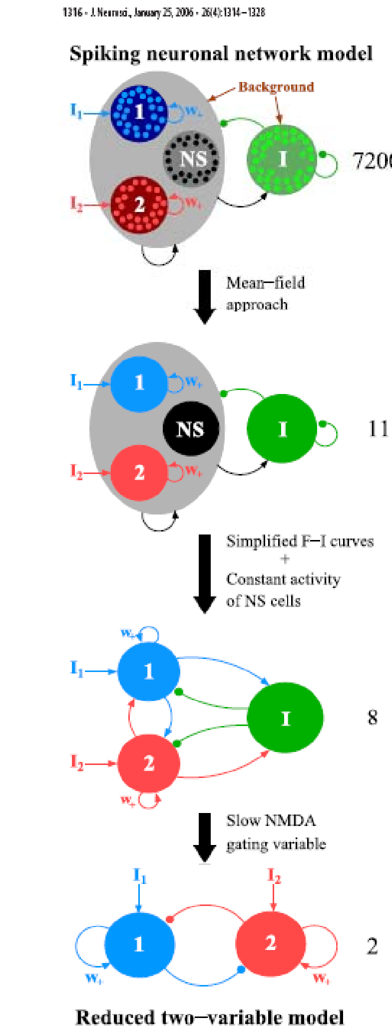
Mean Field Approaches

Biological ⟷ Computational



In vivo

Tsodyks, et al. 1998

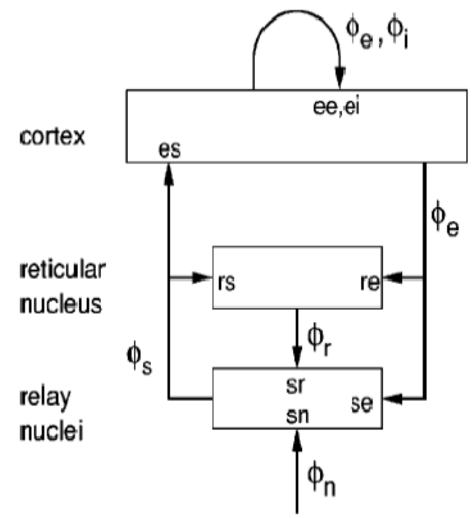


Wong and Wang, 2006

In silica

Robinson's Cortico-Thalamocortical Model

- Specifically models EEG/ECoG
- Delay-Dependent (DDE) and linearized PDE → ODE
- 18 physiologically-based, experimentally-verified parameters
- Parametric decoupling of compartments
- Extensible – more complicated configurations may be constructed across distinct brain regions
- Captures many EEG-specific features in linear dynamics (Breakspear, DeBellis)



A linear resonator model is the simplest instantiation for a stimulus-triggered rise-to-threshold

Robinson Model of Neural Population Dynamics

(Historical development via Wilson & Cowan, Freeman, Amari, Haken, Nunez, and Wright & Liley)

Nonlinear form (Robinson et al., 1997)

$$V_a(t) = \int_{-\infty}^{\infty} L(t-t')P_a(t')dt'$$

$$P_a(t) = \sum_b v_{ab}\phi_b(t)$$

$$\left(\frac{1}{\gamma_a^2}\frac{\partial^2}{\partial t^2} + \frac{2}{\gamma_a}\frac{\partial}{\partial t} + 1 - r_a^2\nabla^2\right)\phi_a(\mathbf{r},t) = Q\left(V_a(\mathbf{r},t)\right)$$

$$L(u) = \frac{\alpha\beta}{\beta-\alpha}\left(e^{-\alpha u} - e^{-\beta u}\right)$$

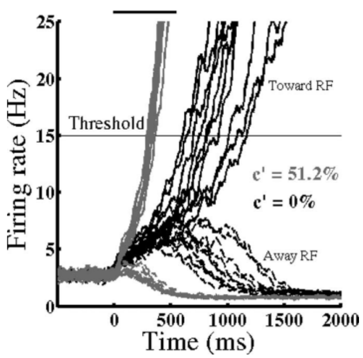
$$Q(x) = \frac{Q_{\max}}{1+e^{\frac{-(V(x)-\theta)}{\sigma}}}$$

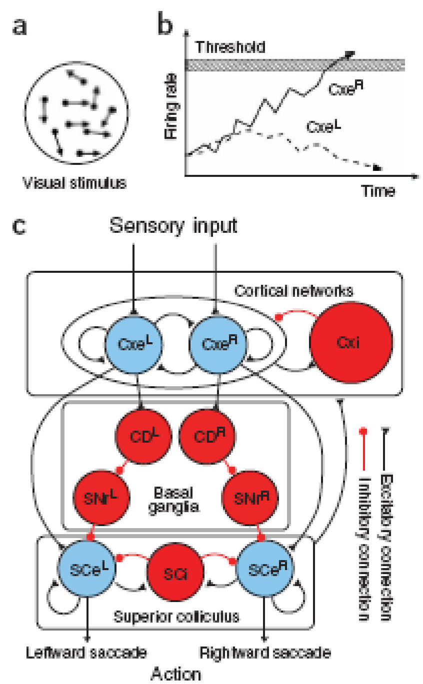
Space-clamped, Linearized form (Robinson et al., 2005)

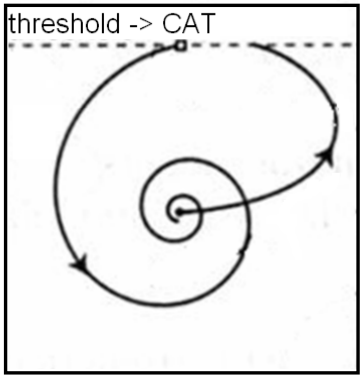
$$D_\alpha V_a(\mathbf{r},t) = \sum_b N_{ab}s_{ab}\phi_b(\mathbf{r},t-\tau_{ab})$$

$$D_\alpha = \frac{1}{\alpha\beta}\frac{d^2}{dt^2} + \left(\frac{1}{\alpha}+\frac{1}{\beta}\right)\frac{d}{dt} + 1$$

Thresholds and Bifurcations







Fuzzy Pole Math

$$D_\alpha V_a(\mathbf{r},t) = \sum_b N_{ab} s_{ab} \phi_b(\mathbf{r}, t - \tau_{ab})$$

$$D_\alpha = \frac{1}{\alpha\beta}\frac{d^2}{dt^2} + \left(\frac{1}{\alpha} + \frac{1}{\beta}\right)\frac{d}{dt} + 1$$

Fuzzy Pole Math

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In time domain, we reduce the Robinson ODE to

$$\frac{d^2 V_a}{dt^2} + (\alpha + \beta)\frac{dV_a}{dt} + \alpha\beta V_a = A\alpha\beta e^{-\gamma t} + \ldots$$

Fuzzy Pole Math

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The Laplace-transformed equation is

$$(s+\alpha)(s+\beta)V_a(s) = \frac{A\alpha\beta}{(s+\gamma)} + \ldots$$

Fuzzy Pole Math

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For $\gamma=\alpha$ (double pole), the $Ae^{-\gamma t}$ RHS forcing function yields a steady state solution with a **Real part:**

$$V_a(t) = \frac{A\alpha\beta}{\beta-\alpha} t \cos \alpha t$$

Fuzzy Pole Math

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The near-resonant "fuzzy-pole" expansion for β or $\gamma = \alpha + \varepsilon$ (ε small), yields a transfer function kernel

$$H_0(s) = \frac{1}{(s+\alpha)(s+\alpha+\varepsilon)} \approx \frac{1}{(s+\alpha)^2}\left[1 - \frac{\varepsilon}{(s+\alpha)} + \frac{\varepsilon^2}{(s+\alpha)^2} - \ldots\right]$$

Fuzzy Pole Math

$$D_\alpha V_a(\mathbf{r},t) = \sum_b N_{ab} s_{ab} \phi_b(\mathbf{r}, t-\tau_{ab}) \qquad D_\alpha = \frac{1}{\alpha\beta}\frac{d^2}{dt^2} + \left(\frac{1}{\alpha}+\frac{1}{\beta}\right)\frac{d}{dt} + 1$$

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The real part of the inverse transform of the above equation is then of the form

$$h(t) \approx t\cos\alpha t\left(1-\frac{\varepsilon t}{2}+\frac{(\varepsilon t)^2}{6}-\ldots\right)$$

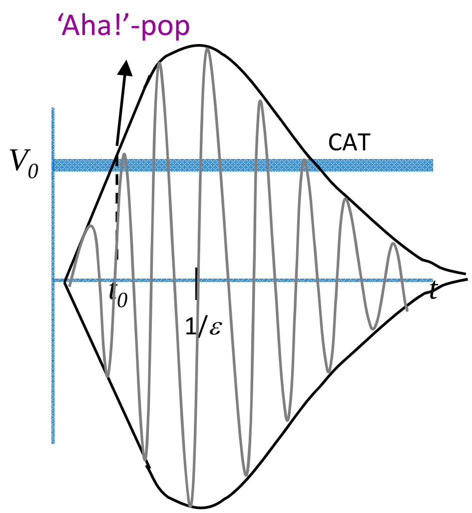
, so that

$$\boxed{V_a(t) = \frac{A\alpha\beta}{\beta-\alpha} t\cos\alpha t\left[1-\frac{\varepsilon t}{2}+\cancelto{0}{\frac{(\varepsilon t)^2}{6}}-\ldots\right]}$$

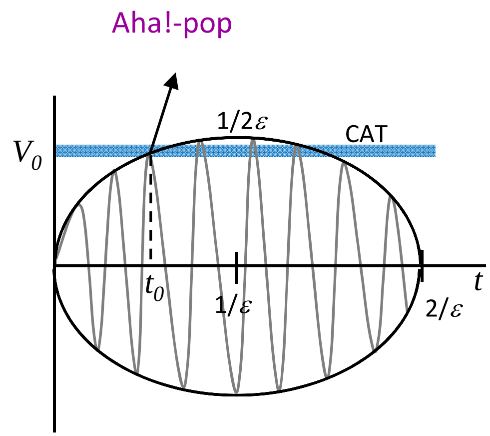
Resonance and “Conscious Access”

$$V_a(t) \approx \frac{A\alpha\beta}{\beta-\alpha} t \cos \alpha t \left[1-\frac{\varepsilon t}{2}\right]$$

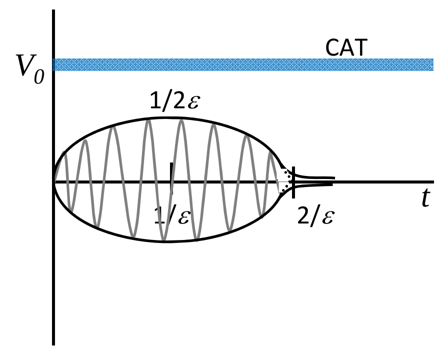
$\gamma = \alpha$; $\varepsilon = 0$
(linear ramp-to-threshold)



$\gamma = \alpha + \varepsilon$; ε small
(threshold crossing)



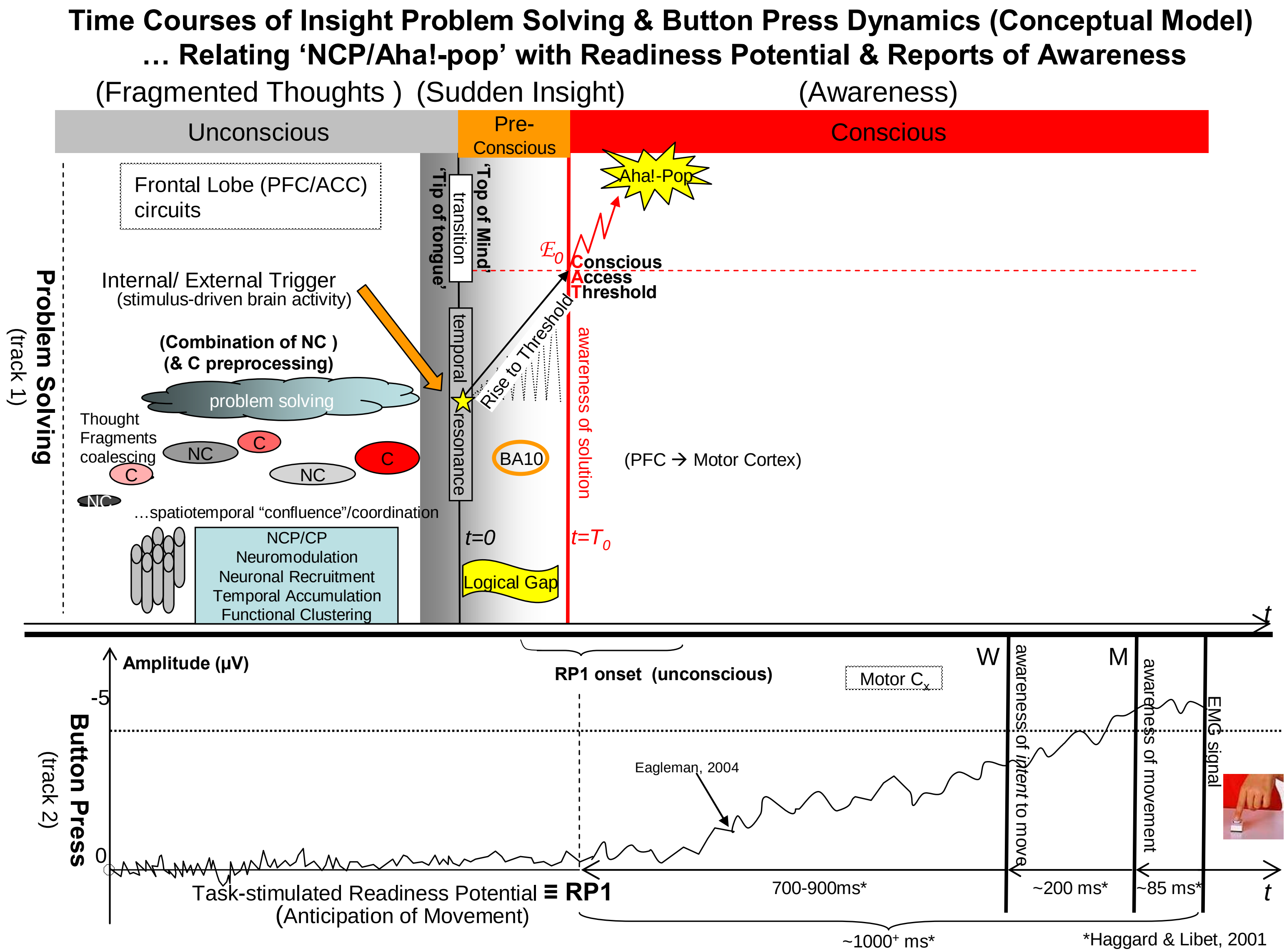
$\gamma = \alpha + \varepsilon$; ε large
(no threshold crossing)



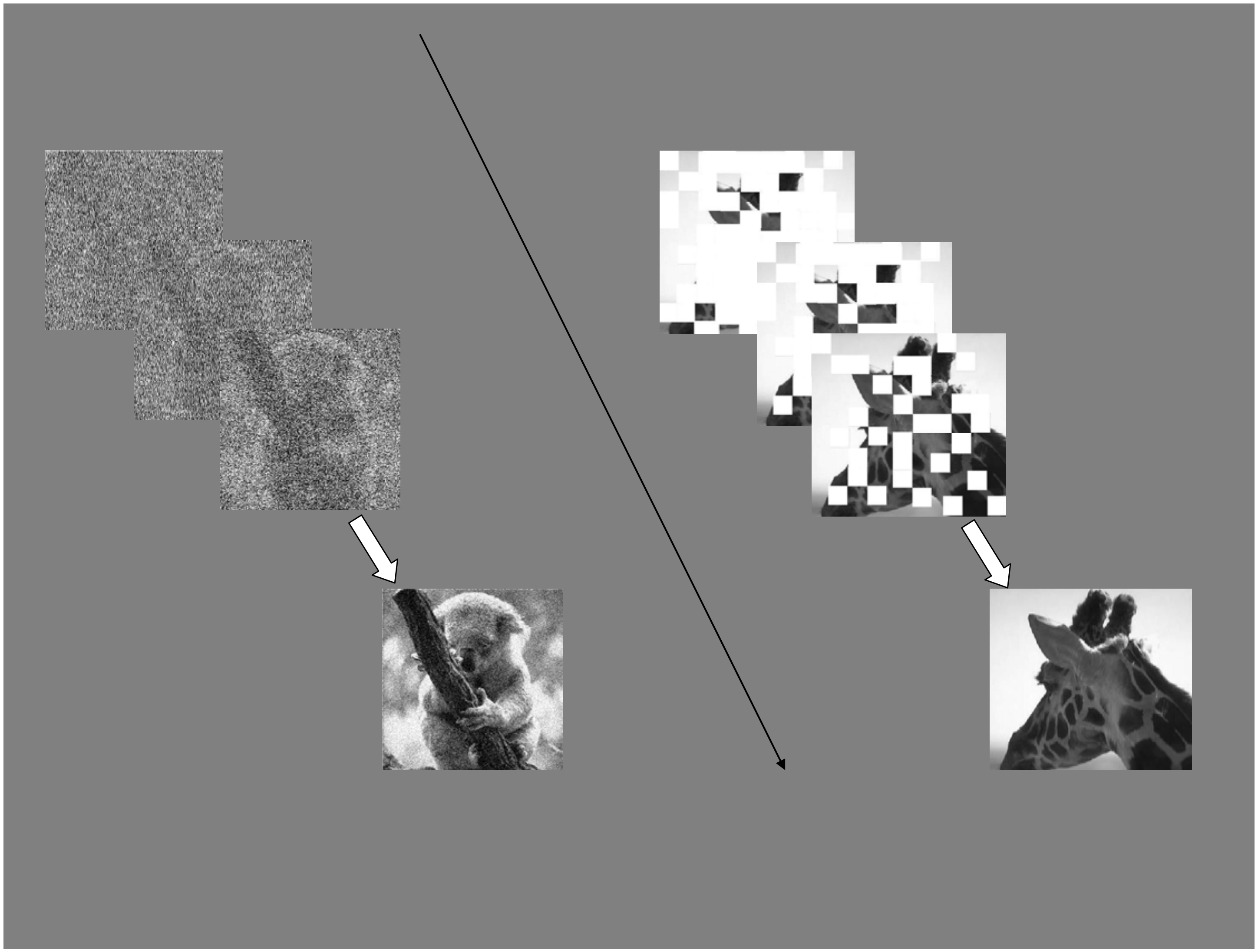
Predictions from Model

- 'Aha!' generates EEG activity with envelopes with linear-exponential envelopes
- EEG gamma band peaks at t_0
- Stimulus 'closeness' represented by ε parameterizes EEG response
- 'Distraction/Noise' attenuates response

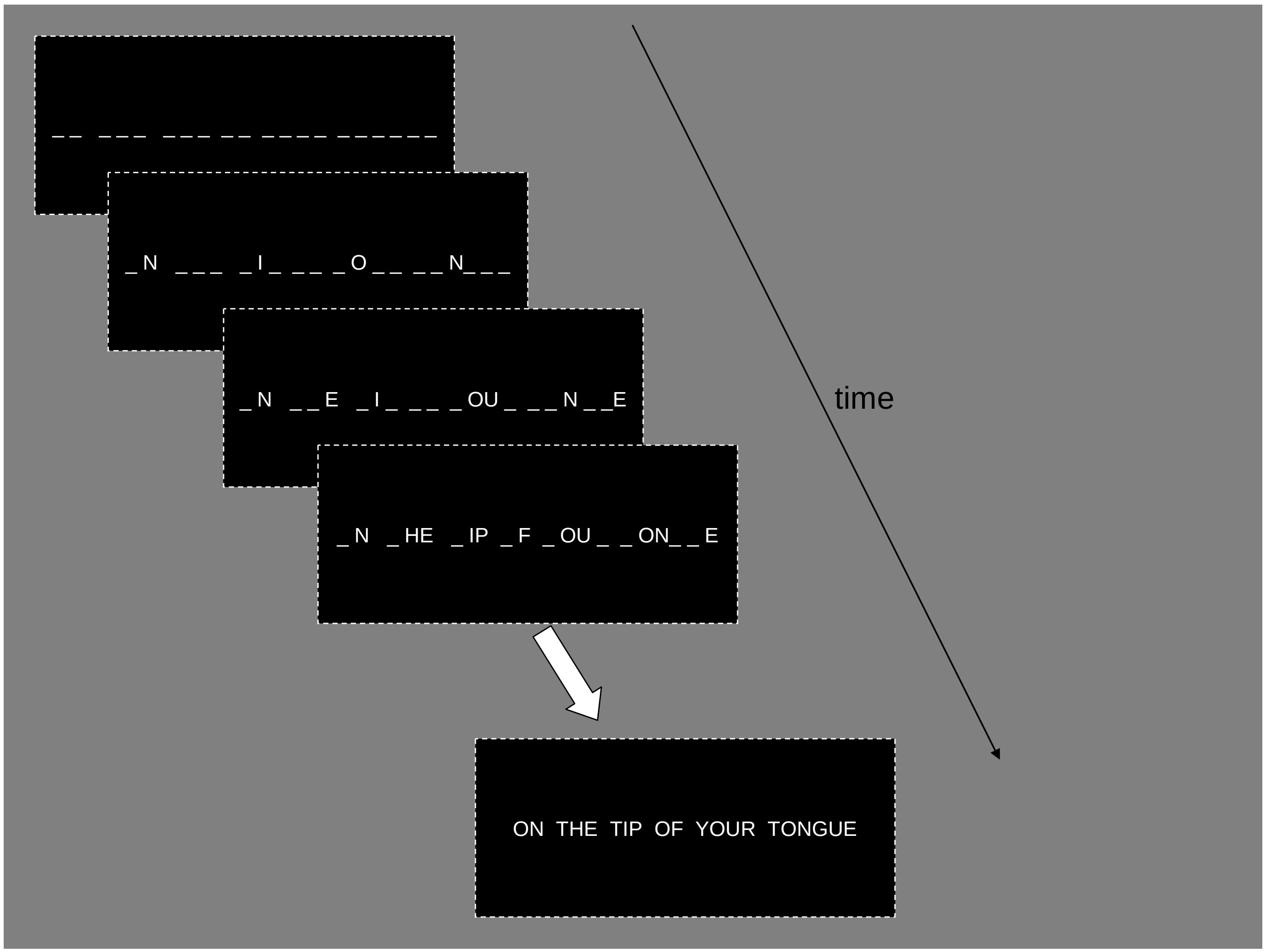
Interleaving of Time Courses



Camouflage Reveal



Phrase Completion



Next Steps

NYU – camouflaged image psychophysics (with N. Rubin)

NYU – computational modeling (with F. Hoppensteadt and J. Rinzel)

UCSD – EEG/ICA dynamics (with S. Makeig)

Thank you